

# Quantum Mechanics

Saturday Morning Physics

Patrick Fox



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"Actually I started out in quantum mechanics, but somewhere along the way I took a wrong turn."

# Please, please ask questions



# My (our?) Challenge

We live in a “classical” world

Our everyday experiences are those of Newton, not Einstein (relativity) or Schrödinger (QM)

*Nothing* prepares you for the *weirdness* of quantum mechanics

# My (our?) Challenge

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--Niels Bohr

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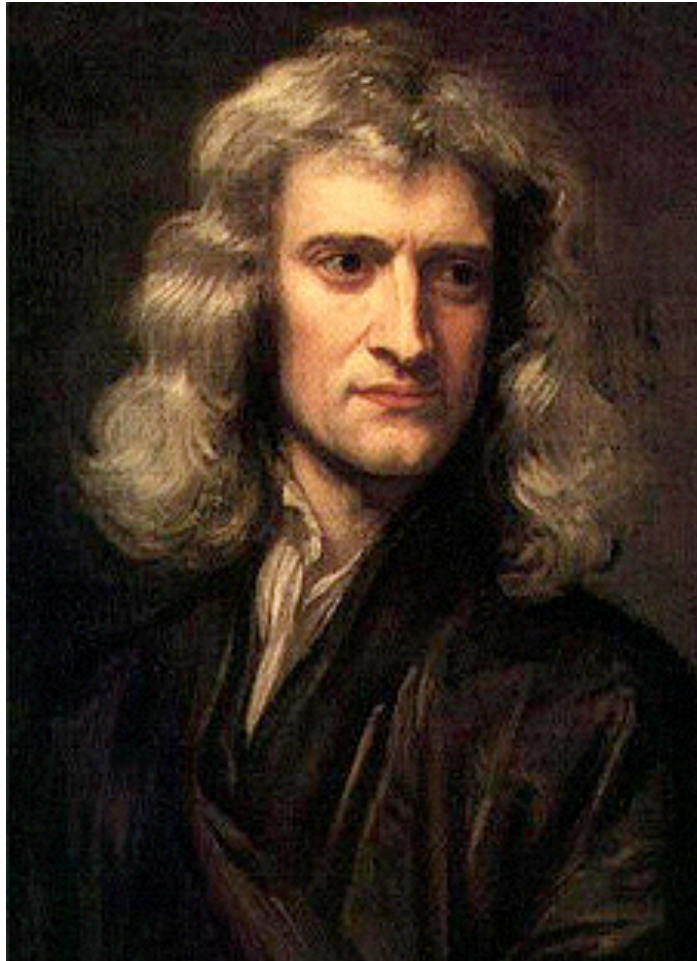
# My (our?) Challenge

How did we discover it?

What is it?

How do we know it is true?

# Classical Physics



Explains all of physics up to ~1900

Deterministic

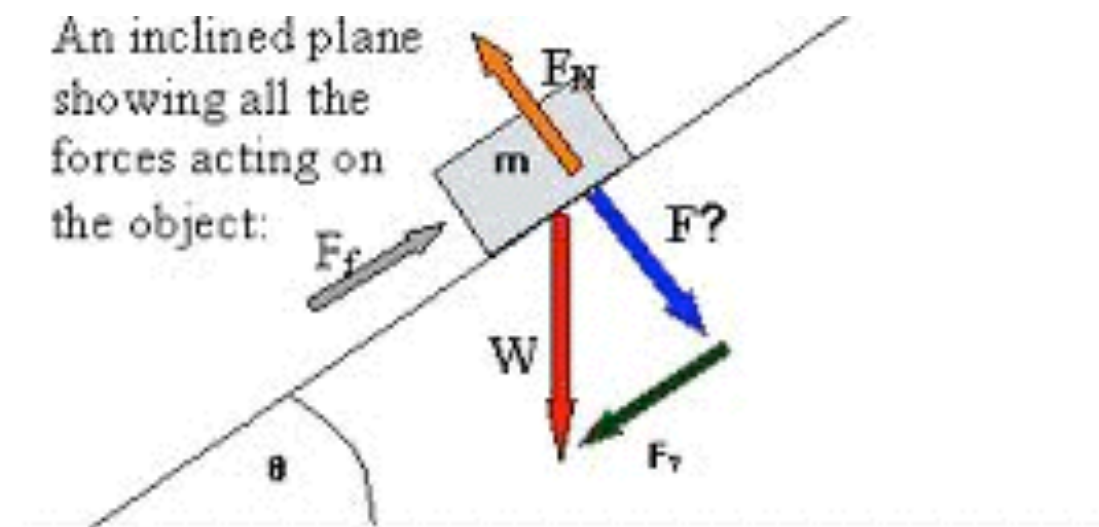
# PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

Autore *J. S. NEWTON*, Trin. Coll. Cantab. Soc. Matheseos  
Professore *Lucasiano*, & Societatis Regalis Sodalis.

IMPRIMATUR.  
S. PEPY S, Reg. Soc. PRÆSES.  
Julii 5. 1686.

LONDINI,  
Jussu Societatis Regiæ ac Typis *Josephi Streater*. Prostat apud  
plures Bibliopolas. Anno MDCLXXXVII.

An inclined plane  
showing all the  
forces acting on  
the object:



$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \epsilon_0$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_B / dt$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \epsilon_0 d\Phi_E / dt$$

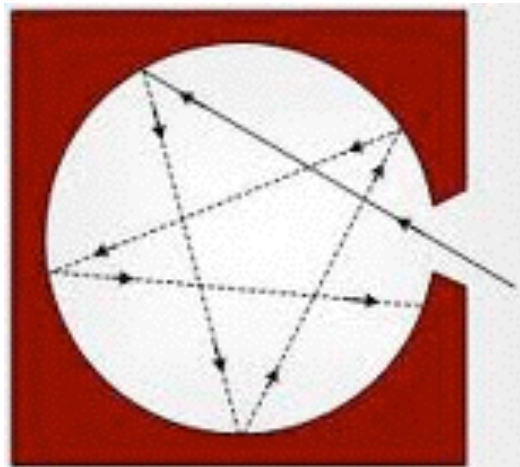
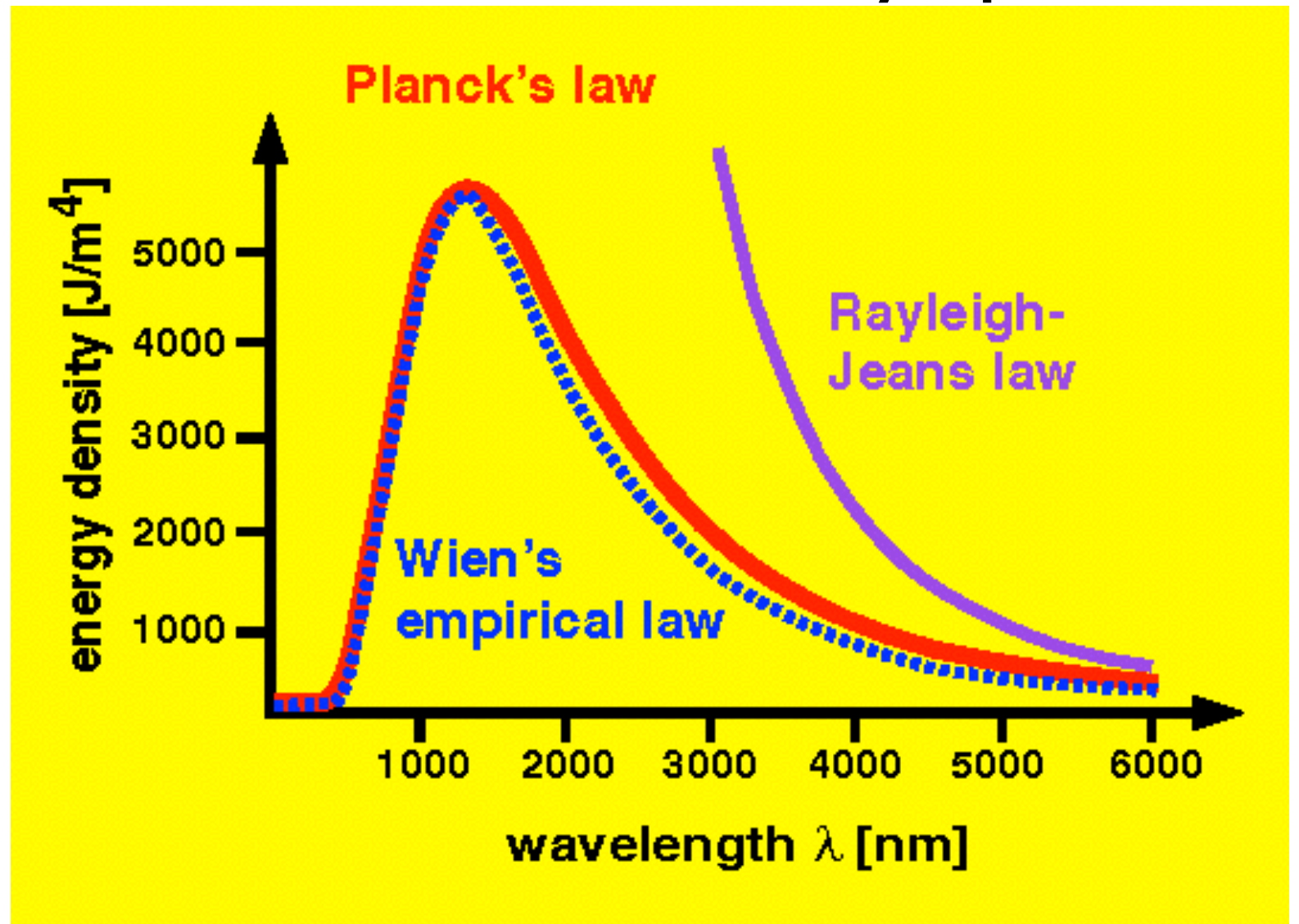


# The beginning of the end of classical physics

## Black Body spectrum



Max Planck



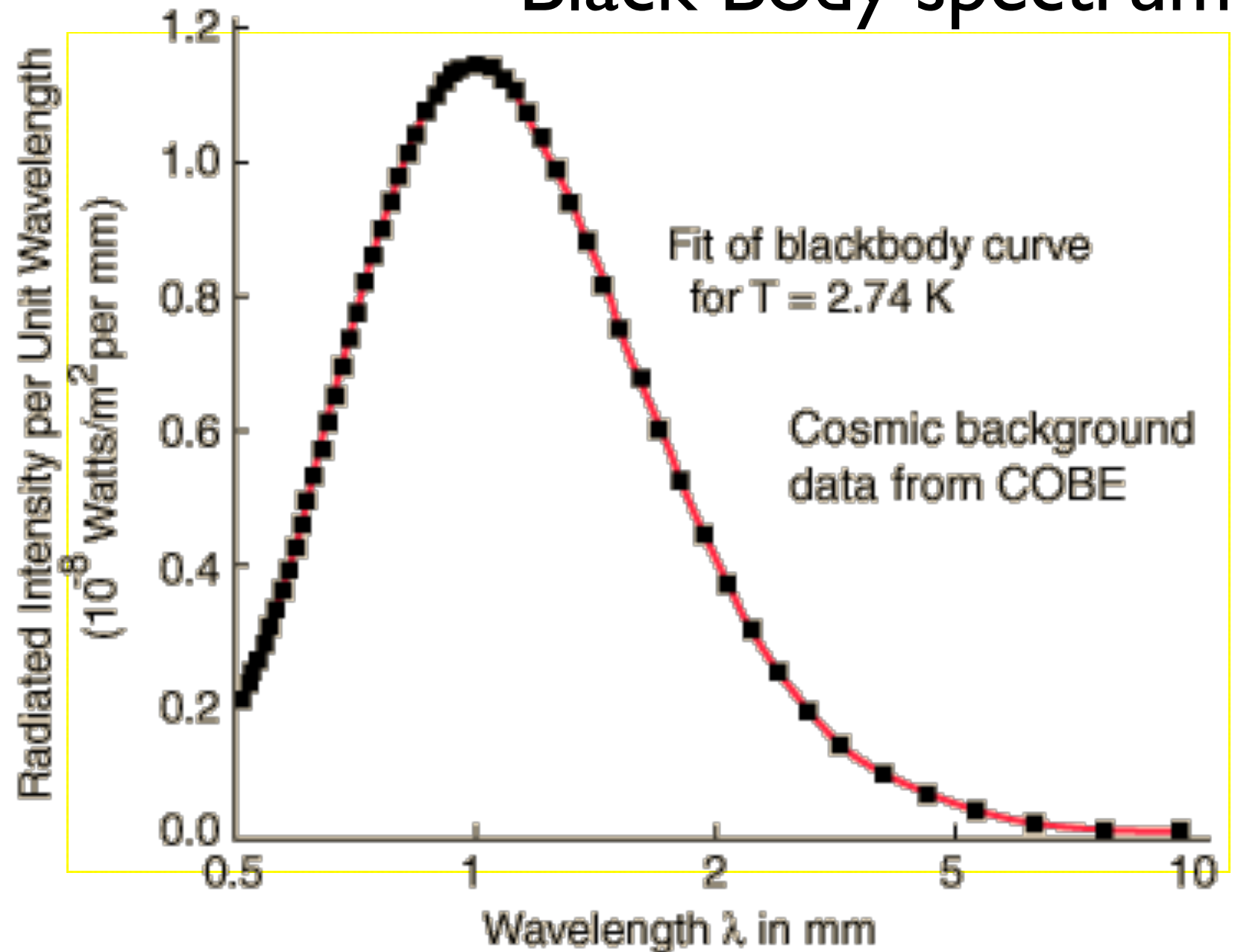
Light can only be emitted and absorbed in discrete units of energy, **QUANTA**

# The beginning of the end of classical physics

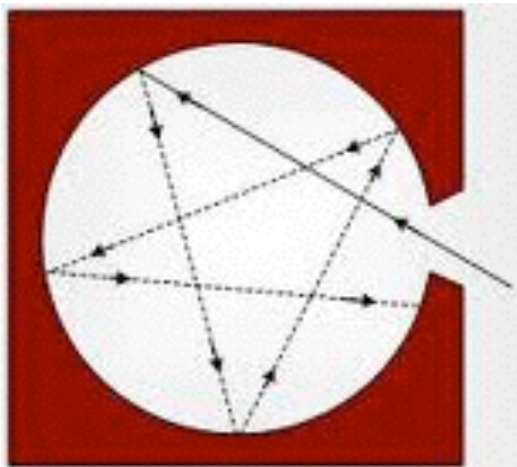
## Black Body spectrum



Max Planck



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# Planck's constant

Light of frequency  $\nu$  can only be emitted and absorbed in units (quanta) of  $h\nu$

$$E = h\nu$$

$$h = 6.626068 \times 10^{-34} m^2 kg/s = 6.626068 \times 10^{-34} Js$$

A new fundamental constant of nature: **Planck's constant**



# Why it took so long to notice quanta

How many quanta of light are emitted from the light bulb above us?

$$E = h\nu$$

$$h = 6.626068 \times 10^{-34} \text{ kg m}^2/\text{s} = 6.62608 \times 10^{-34} \text{ J s}$$



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$$N = \frac{100 \text{ J/s}}{E_q} = \frac{100 \text{ J/s}}{3.3 \times 10^{-19} \text{ J}} \approx 3 \times 10^{20} / \text{s} !!!$$

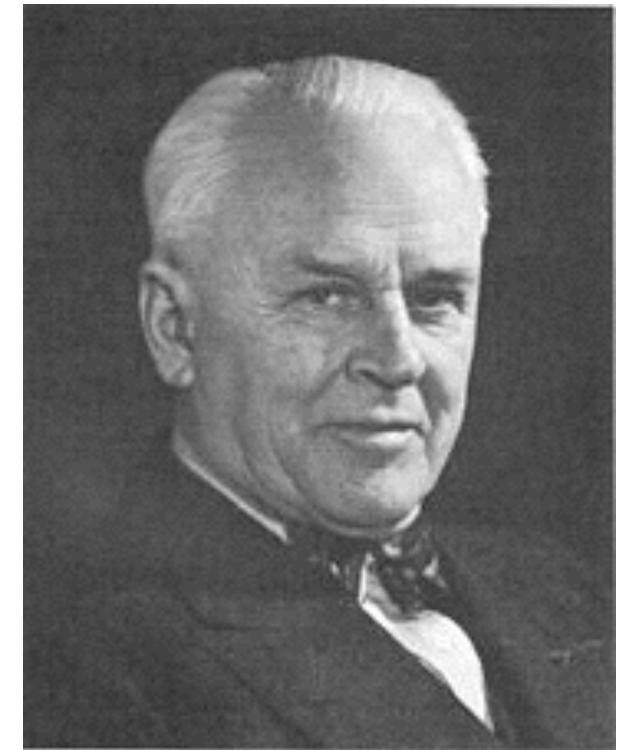
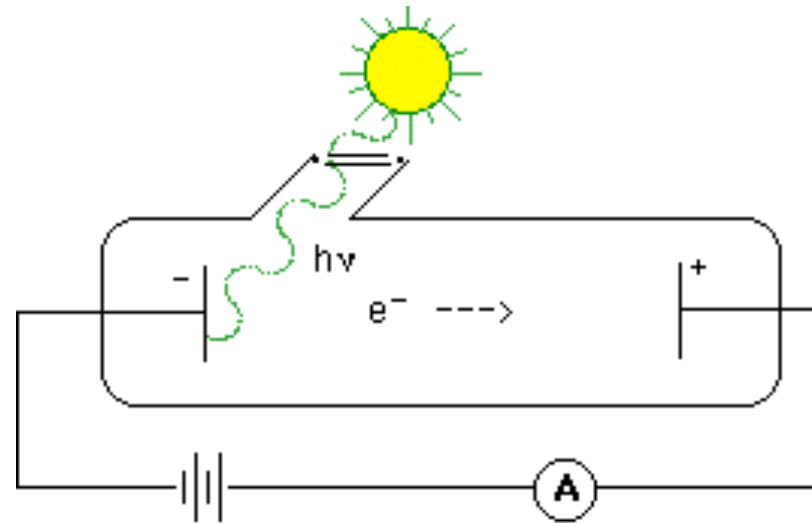


# The end of classical physics

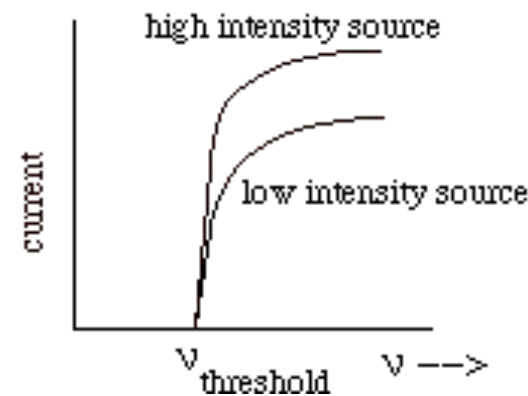


Heinrich Hertz

## Photoelectric effect



R A Millikan



1. Metals emit electrons when irradiated
2. Threshold, depends on frequency

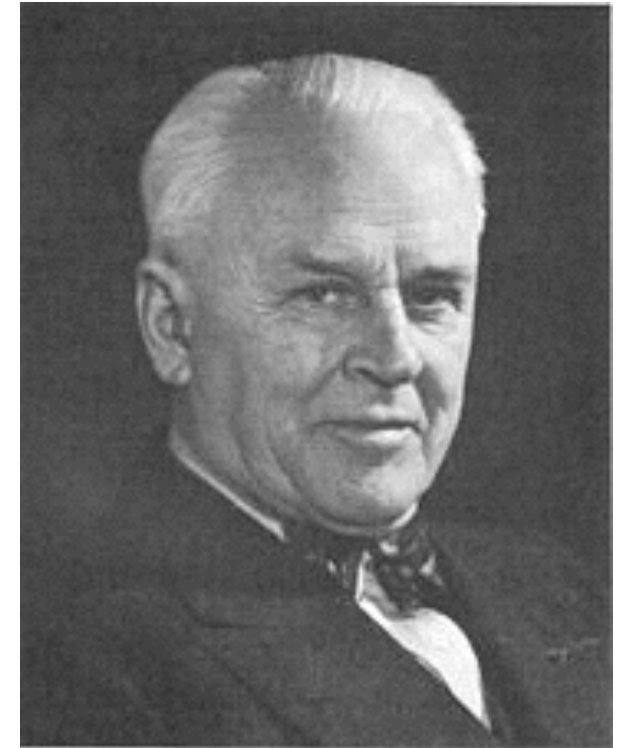
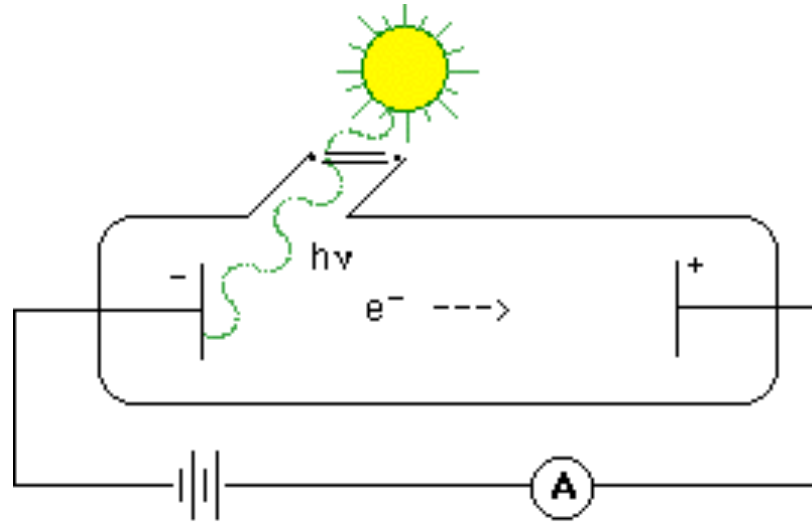
3. Current  $\propto$  intensity
4. Energy  $\propto$  frequency

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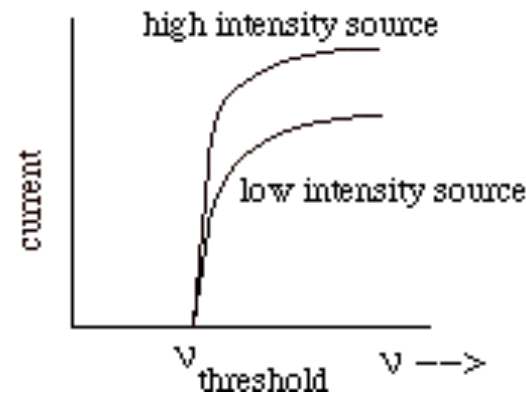


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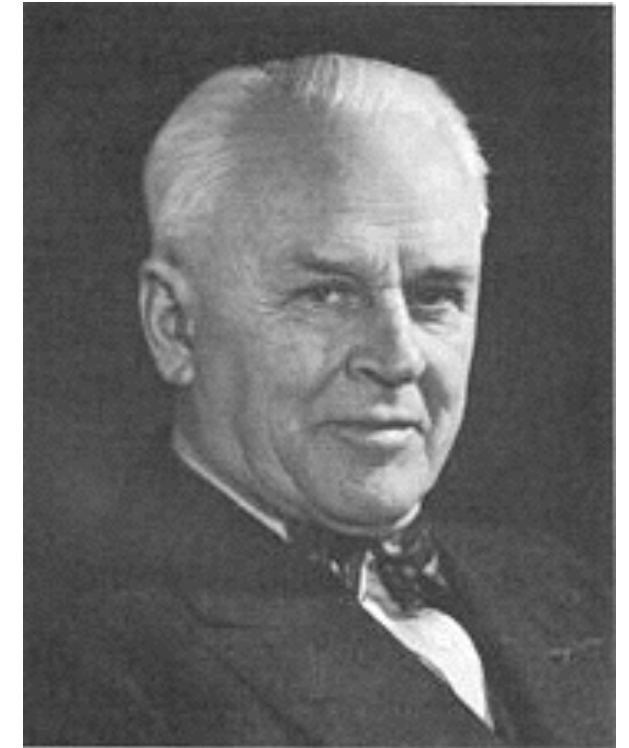
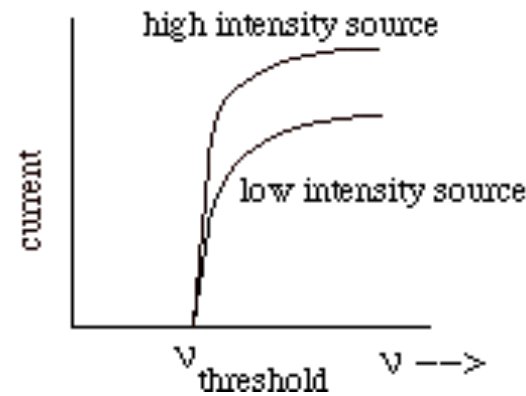
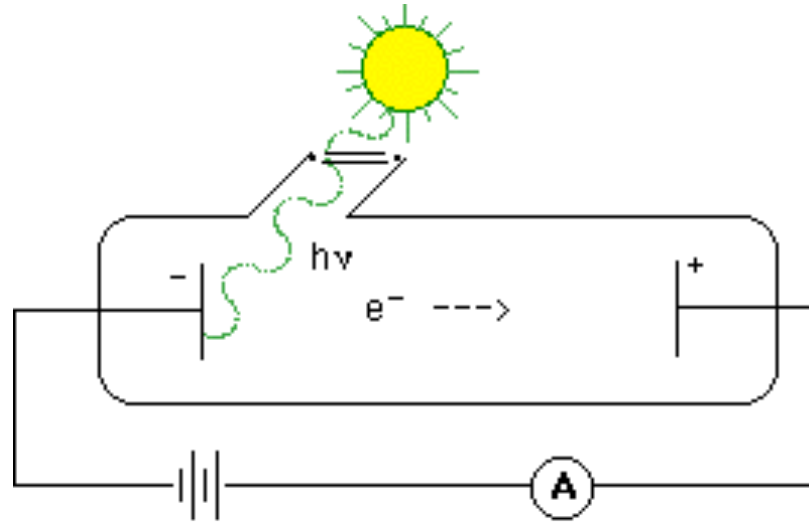


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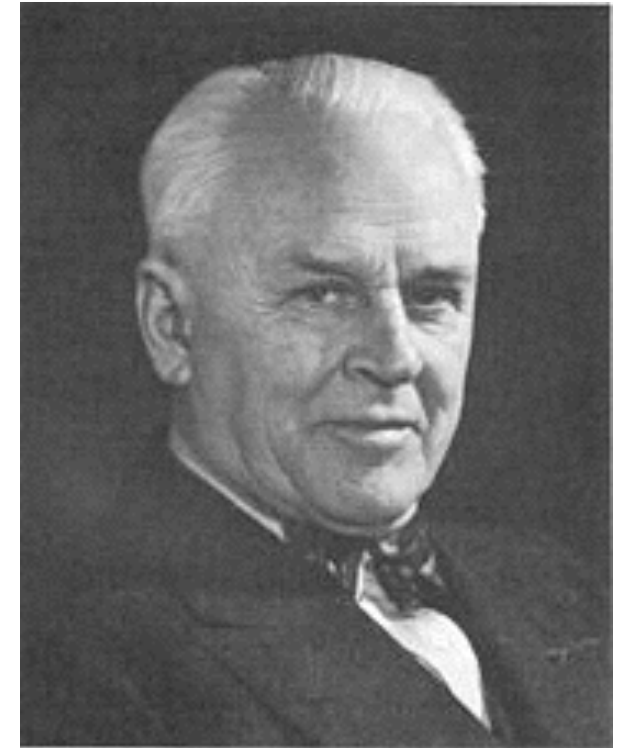
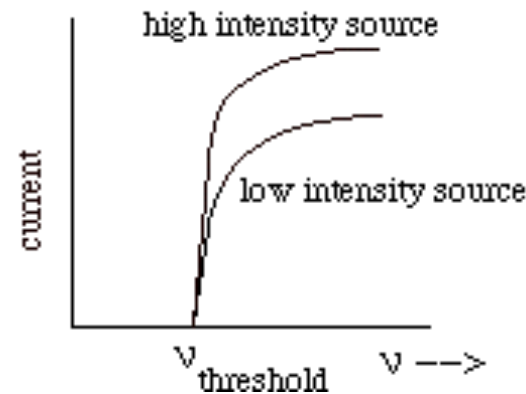
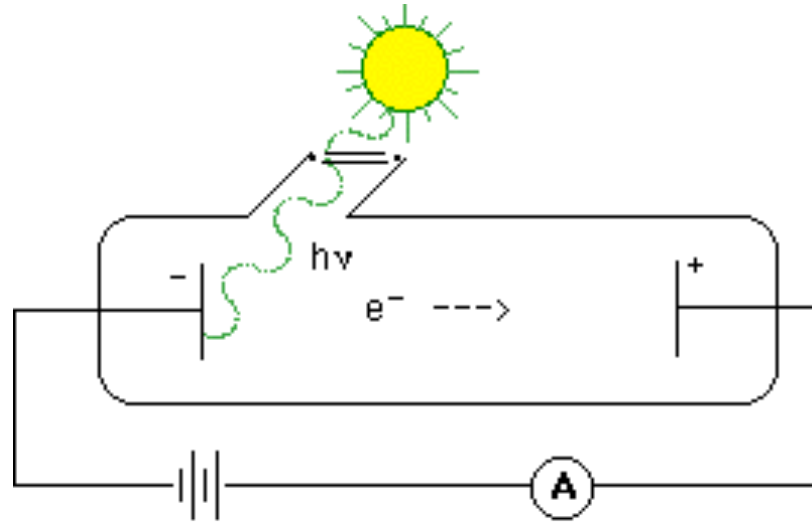
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# The end of classical physics



Heinrich Hertz

## Photoelectric effect



R A Millikan

Photons as waves cannot explain this!



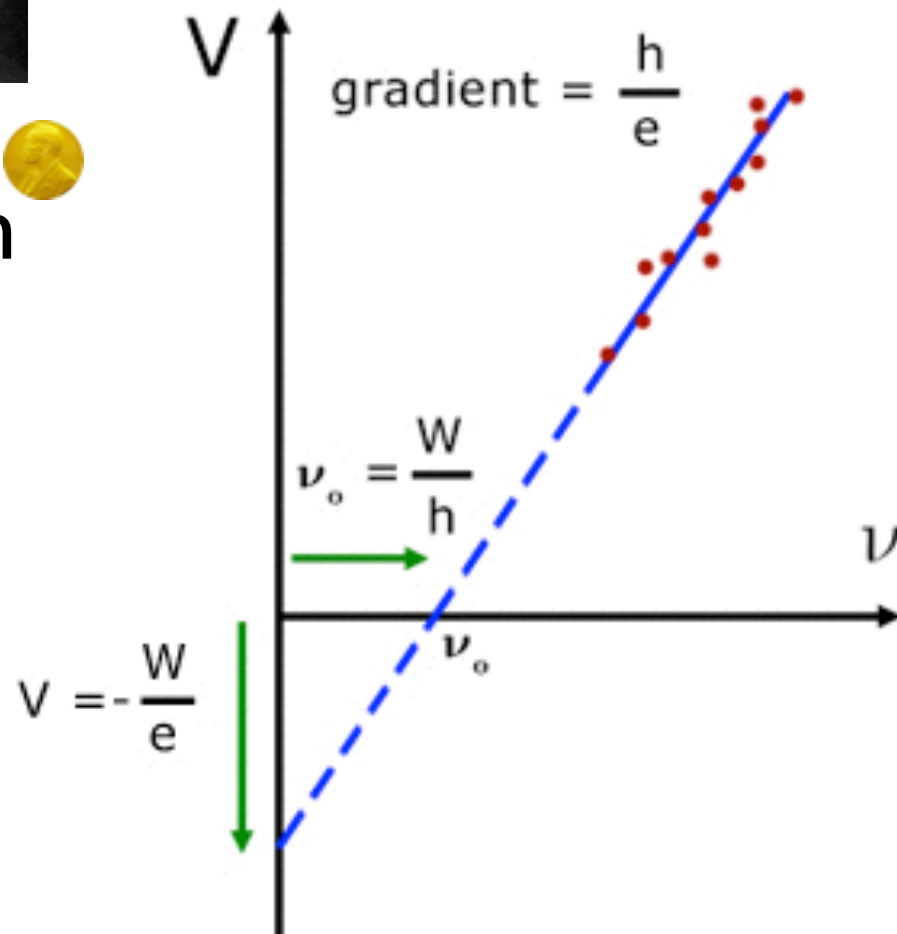
# The end of classical physics



Albert Einstein

Light behaves as if it is discrete bundles of energy (photons) of energy  $h\nu$

$$\frac{1}{2}mv^2 = h\nu - W$$



Explains photoelectric effect

Light is a particle!

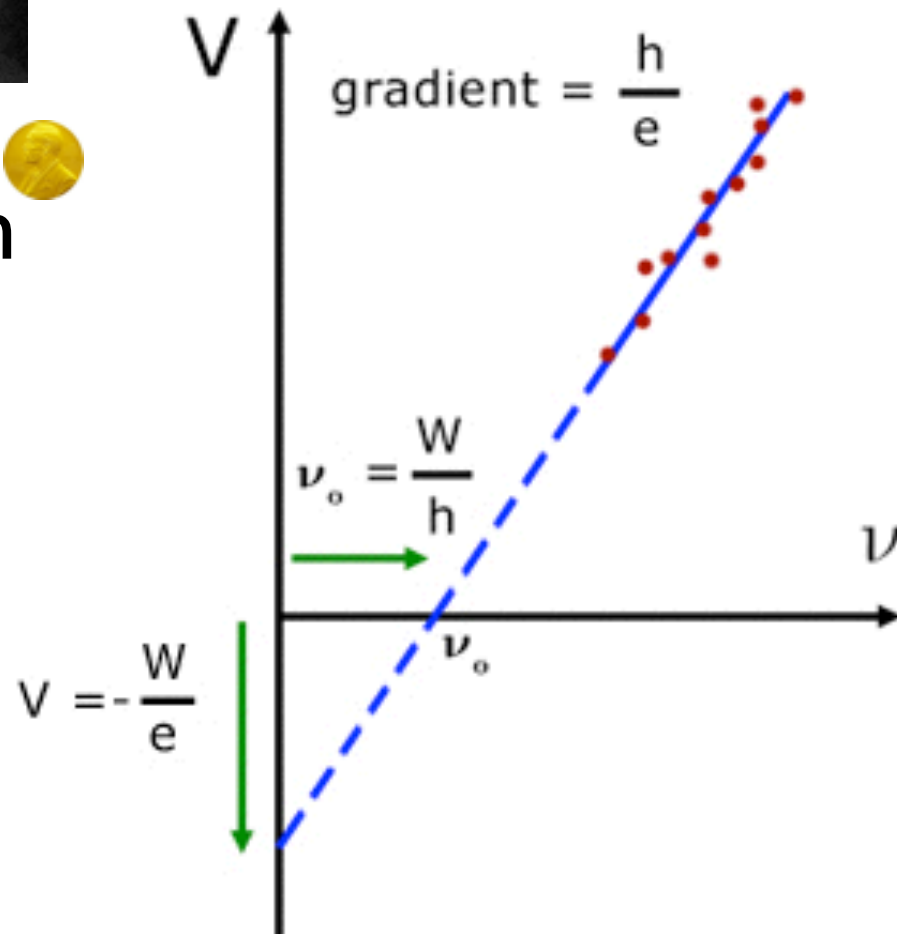
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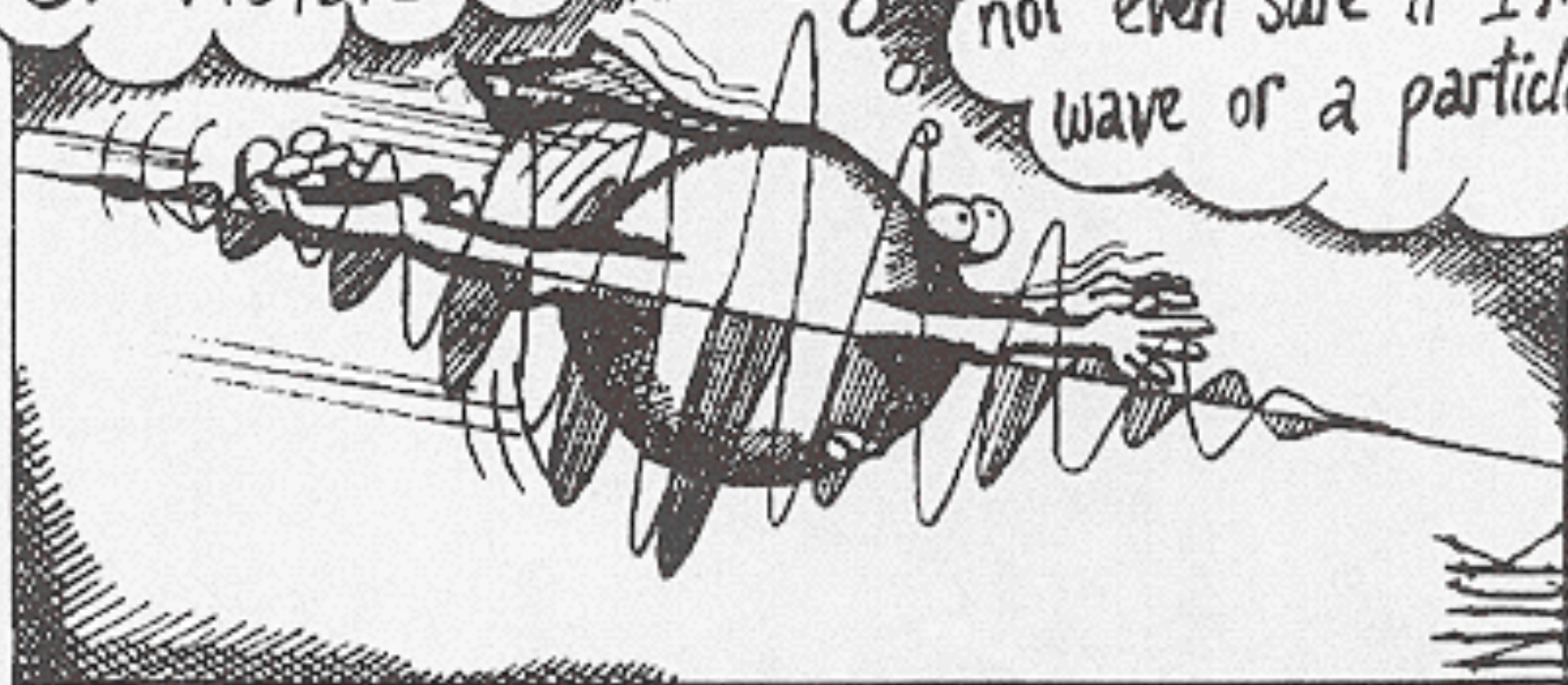


Explains photoelectric effect

Light is a particle!  
and a wave!

Am I an X-ray  
photon...? Or a  
radio photon?  
Or visible?

Oh hell..! Why worry about  
all that again...? I'm  
not even sure if I'm a  
wave or a particle!



= PHOTON SELF-IDENTITY PROBLEMS =

# Wave-particle duality



Louis de Broglie 🏆

If a wave can be a particle, can a particle be a wave?

For light,

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

What is your de Broglie wavelength?



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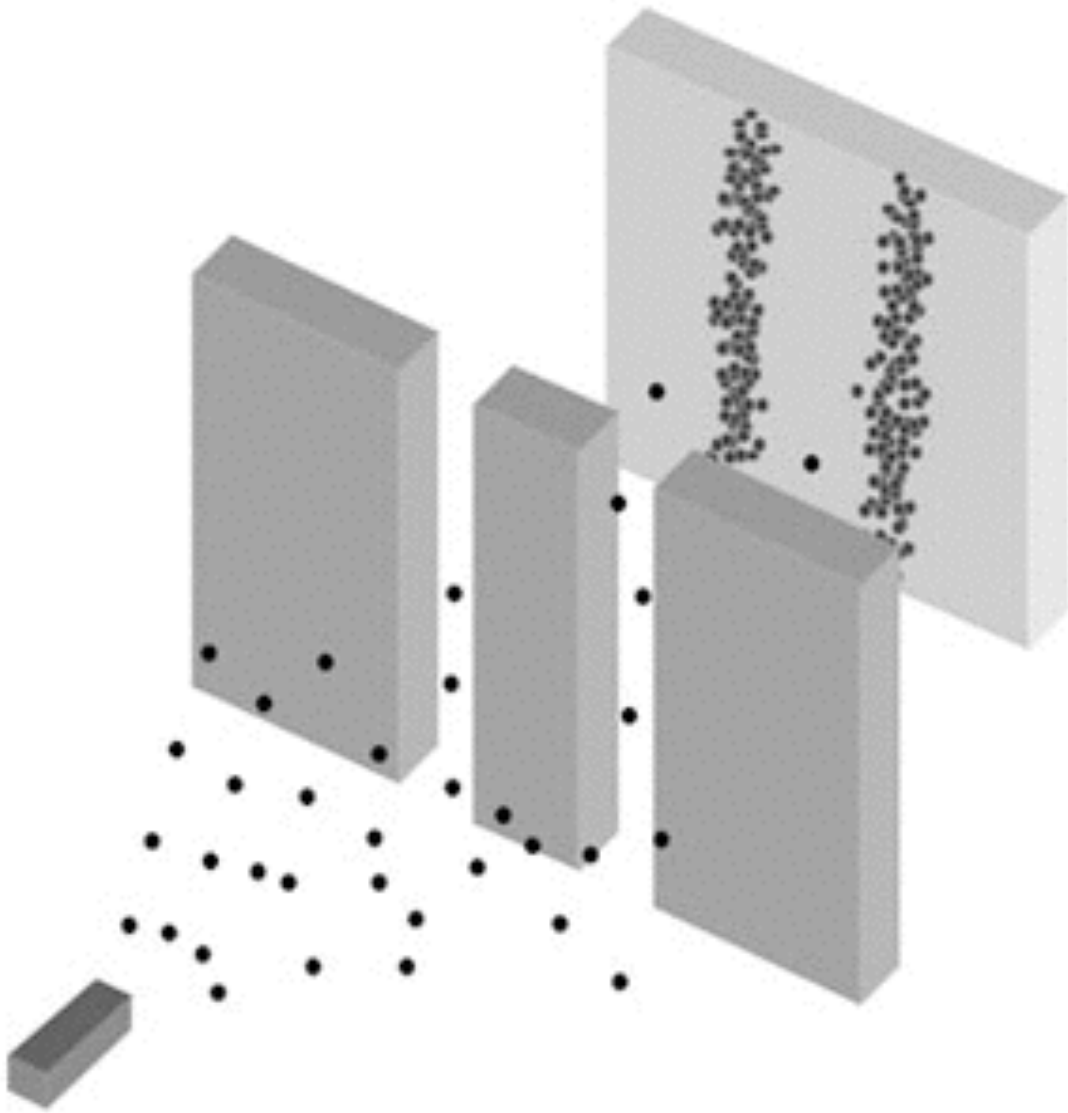
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proton is about

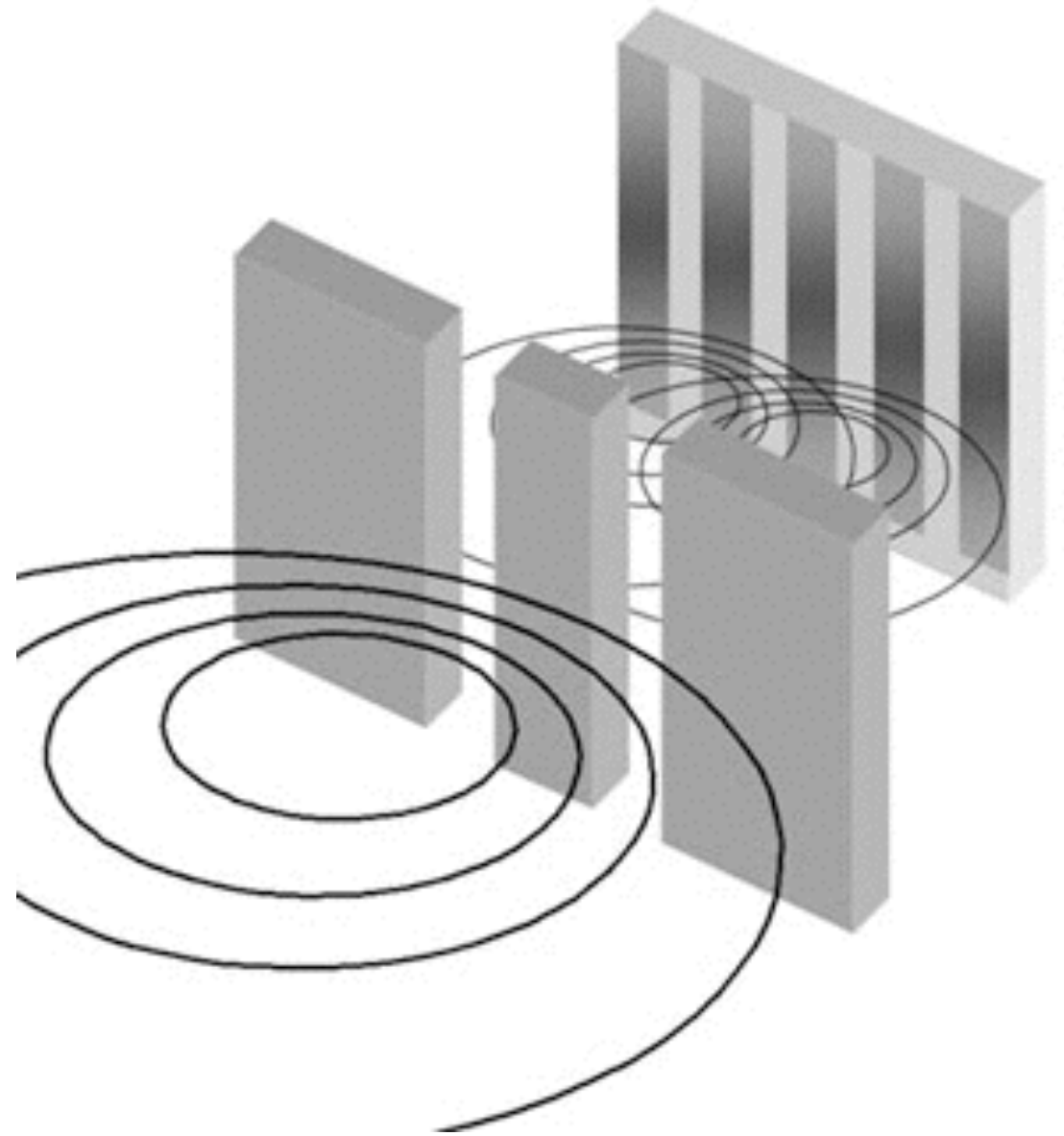
$$10^{-15} m$$



# The double slit experiment

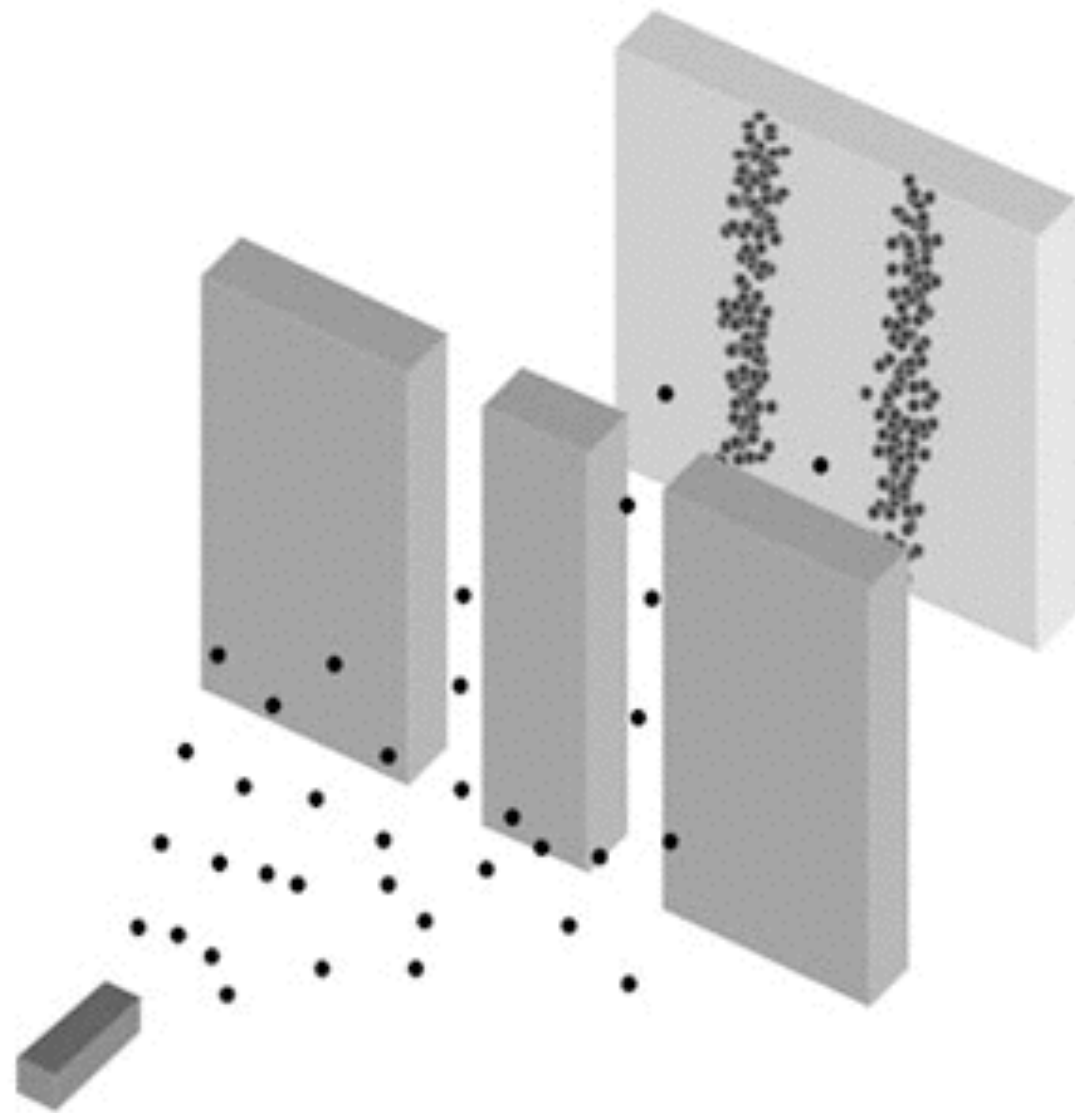


Classical particles, e.g.  
bullets



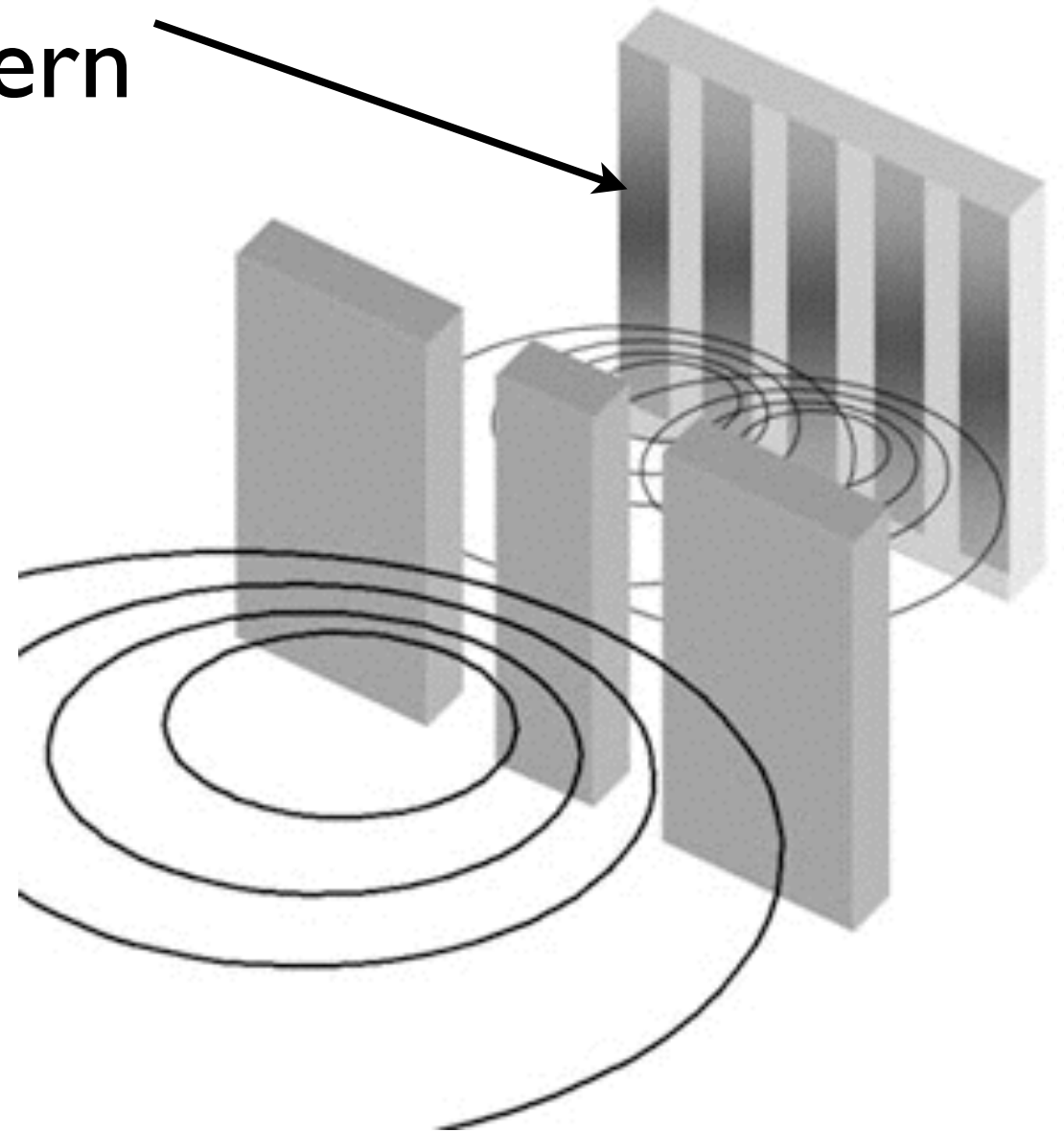
Waves, e.g. on water  
surface

# The double slit experiment

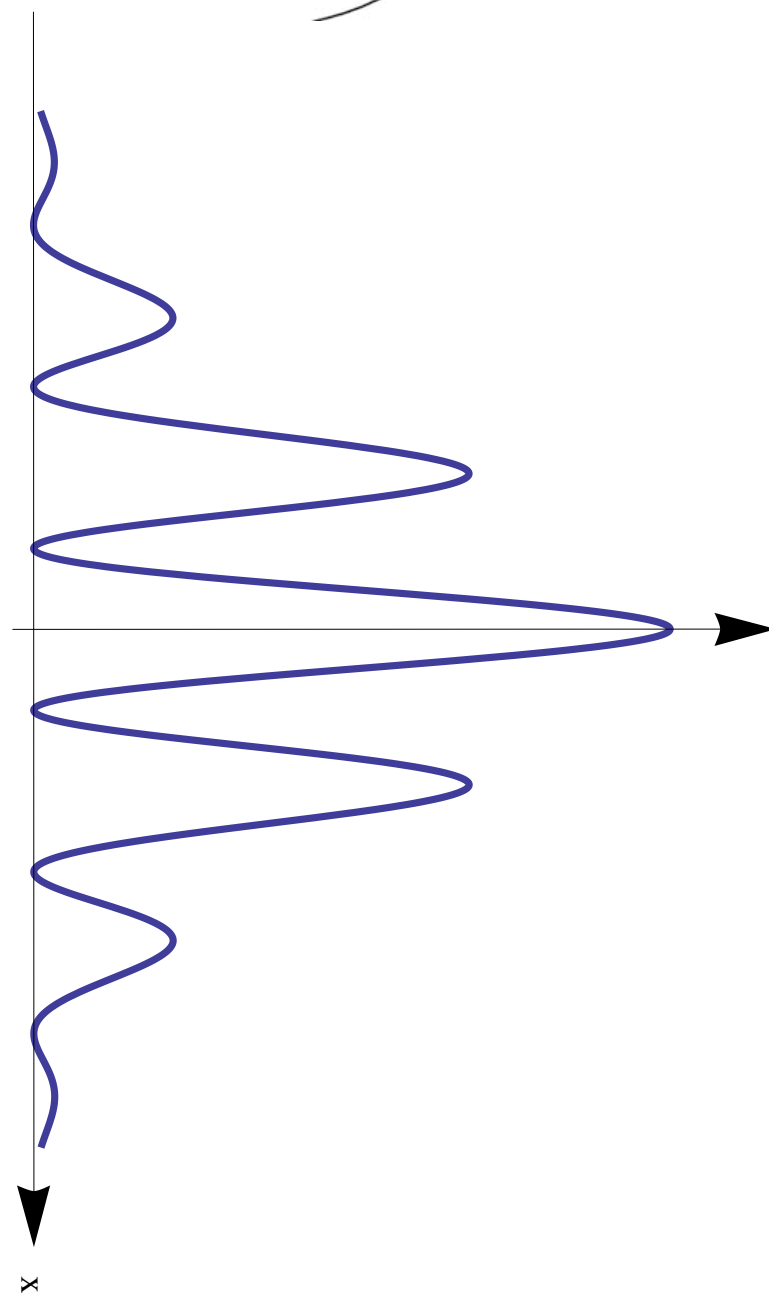
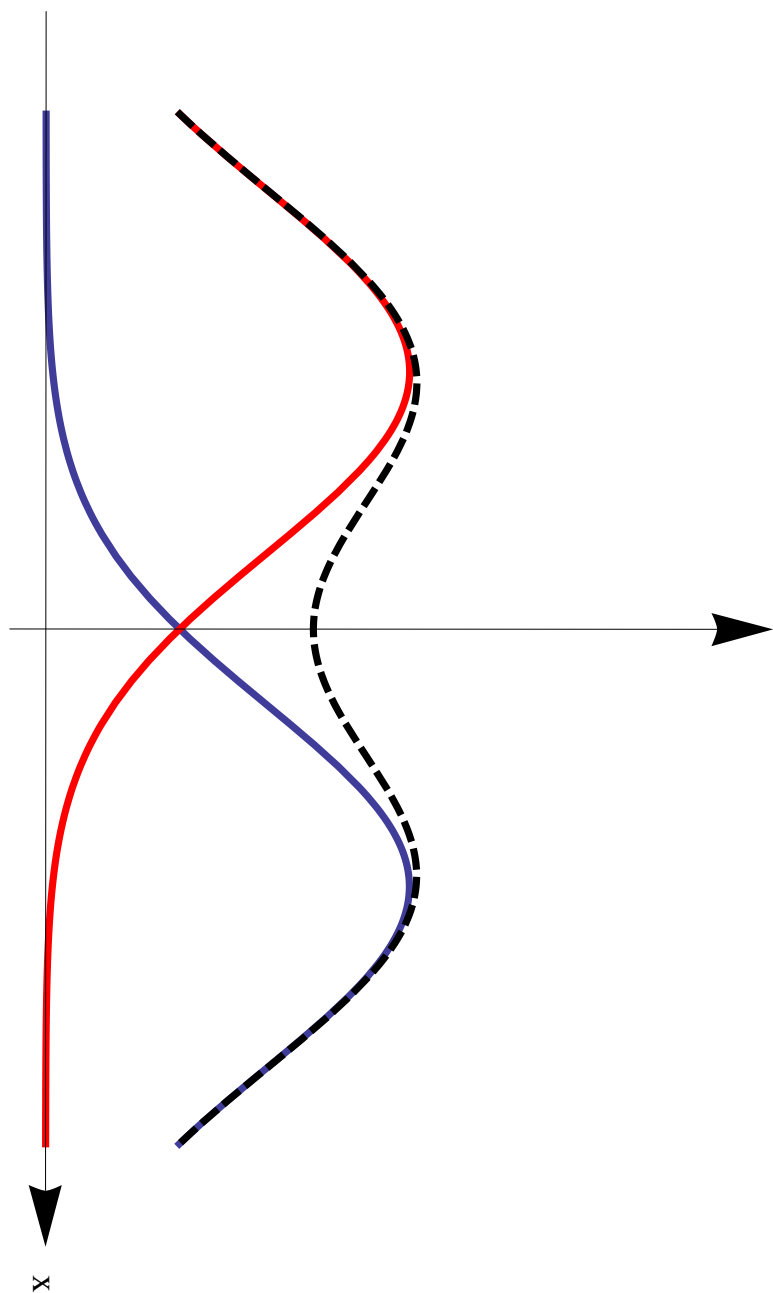
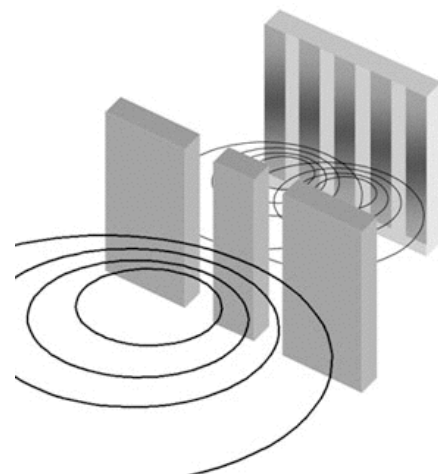
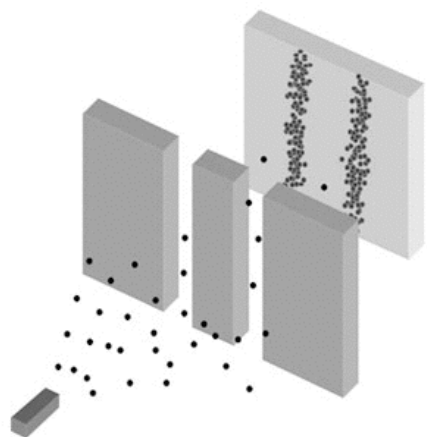


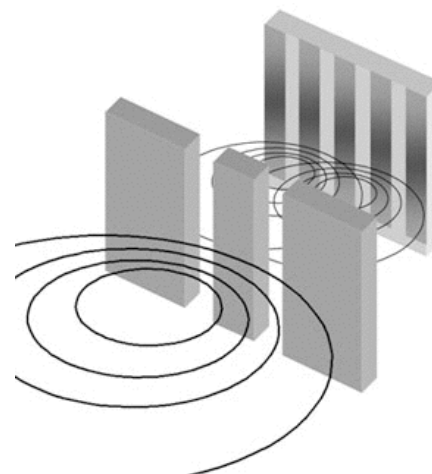
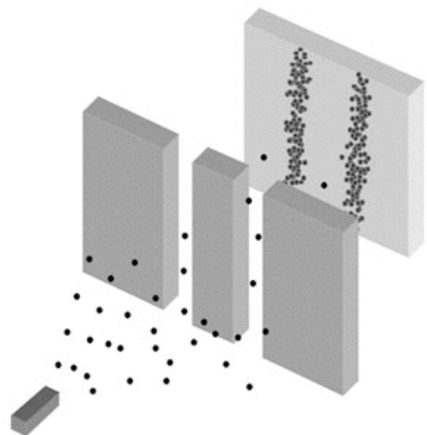
Classical particles, e.g.  
bullets

Interference  
pattern

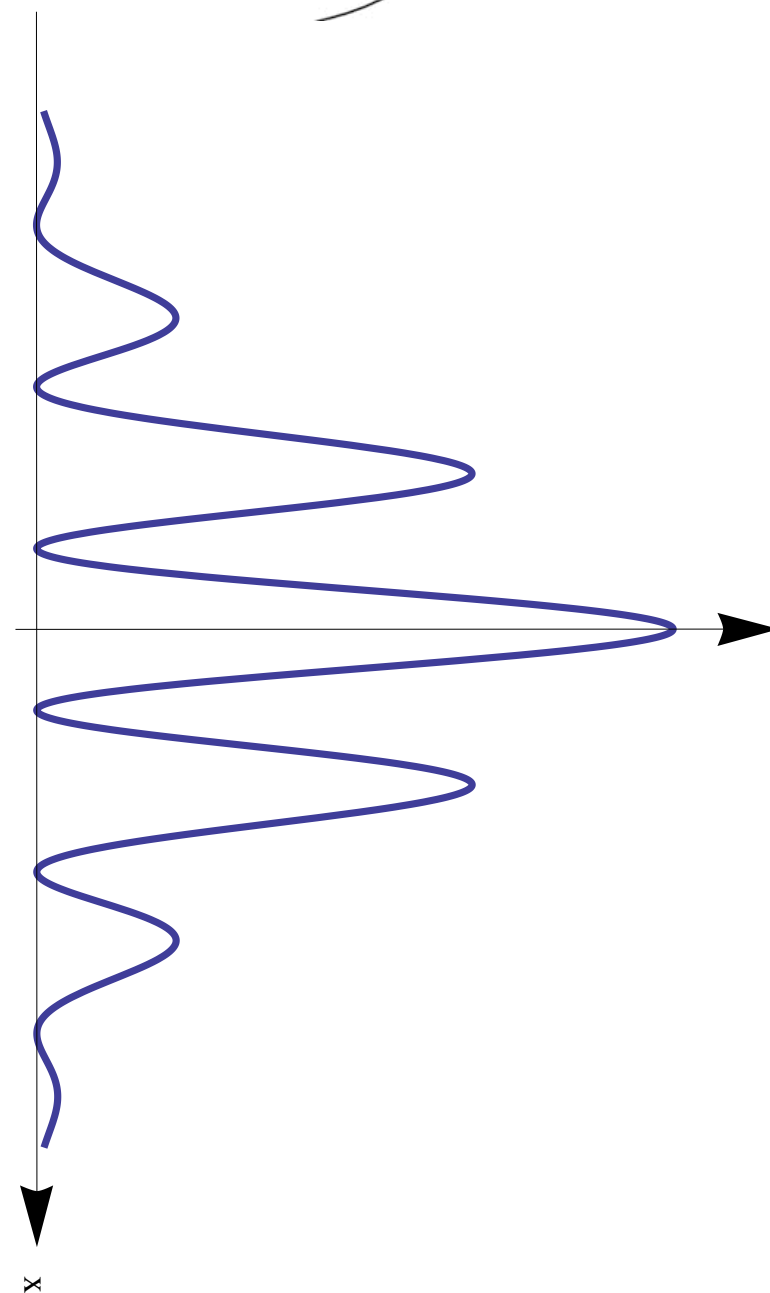
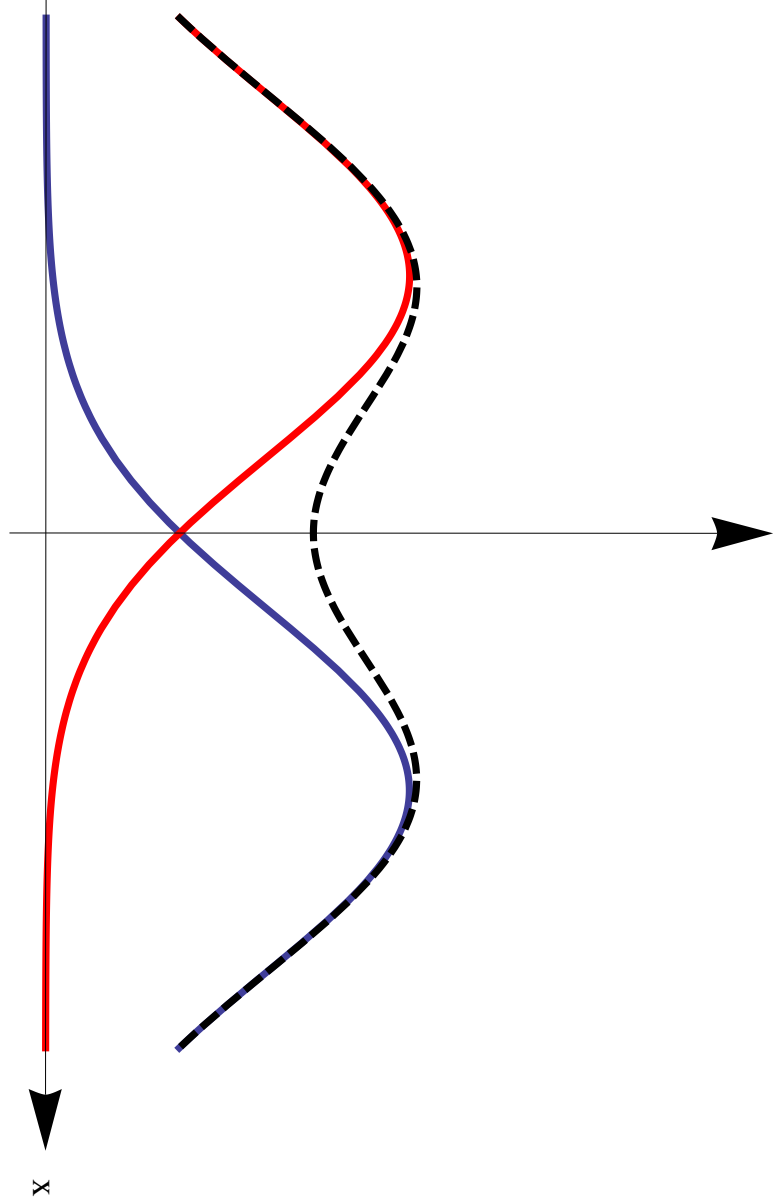


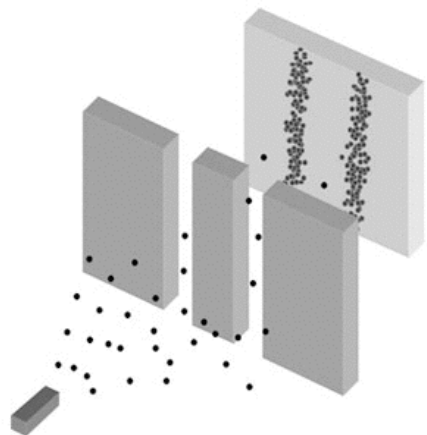
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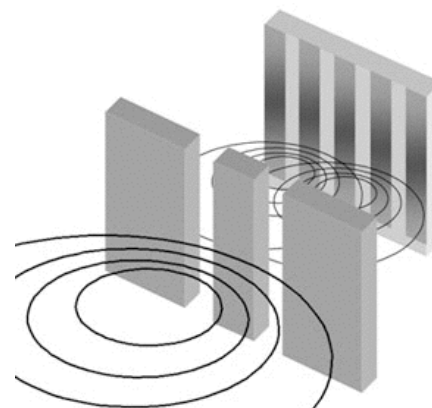
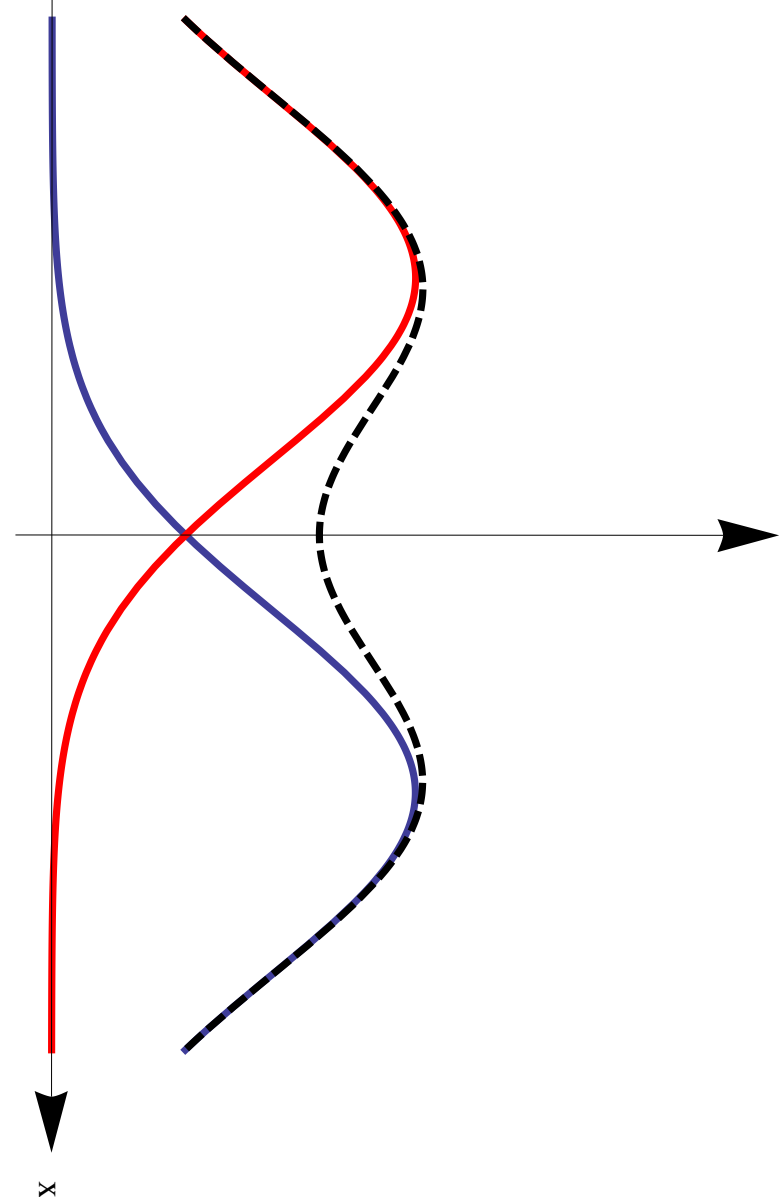


$$P_{12} = |A_1|^2 + |A_2|^2$$

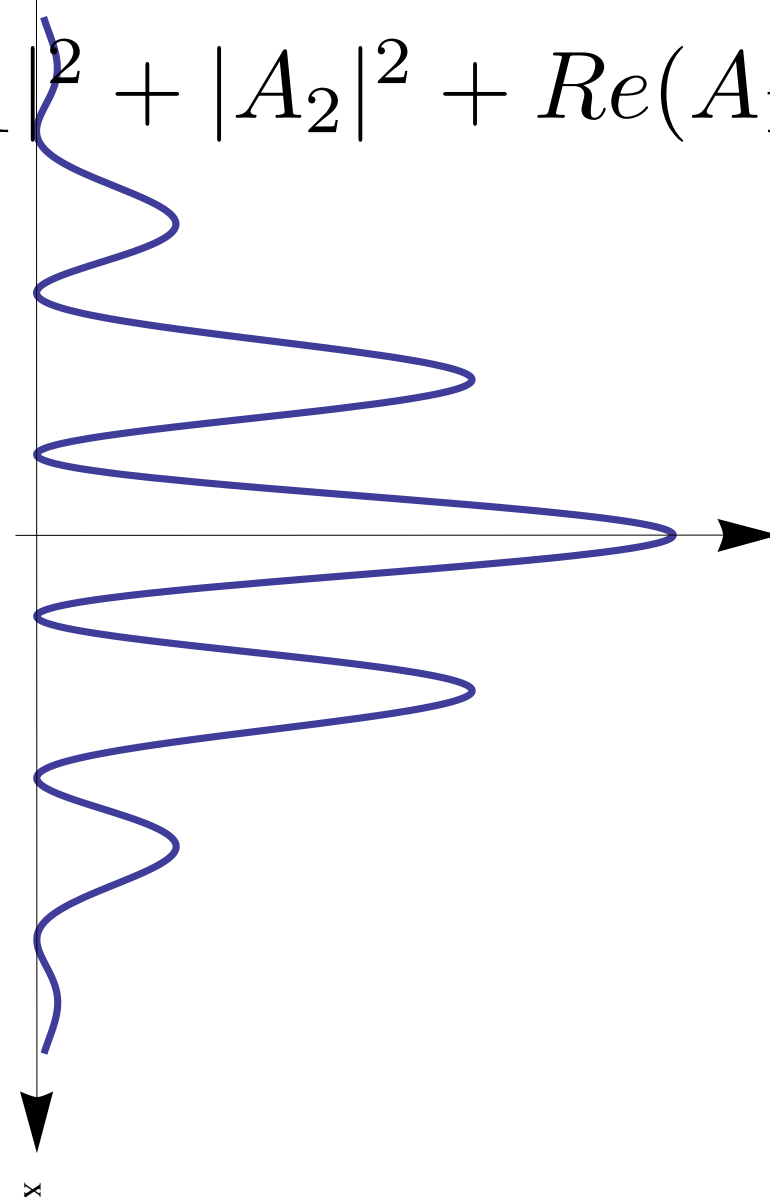




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# The double slit experiment

Repeat with electrons

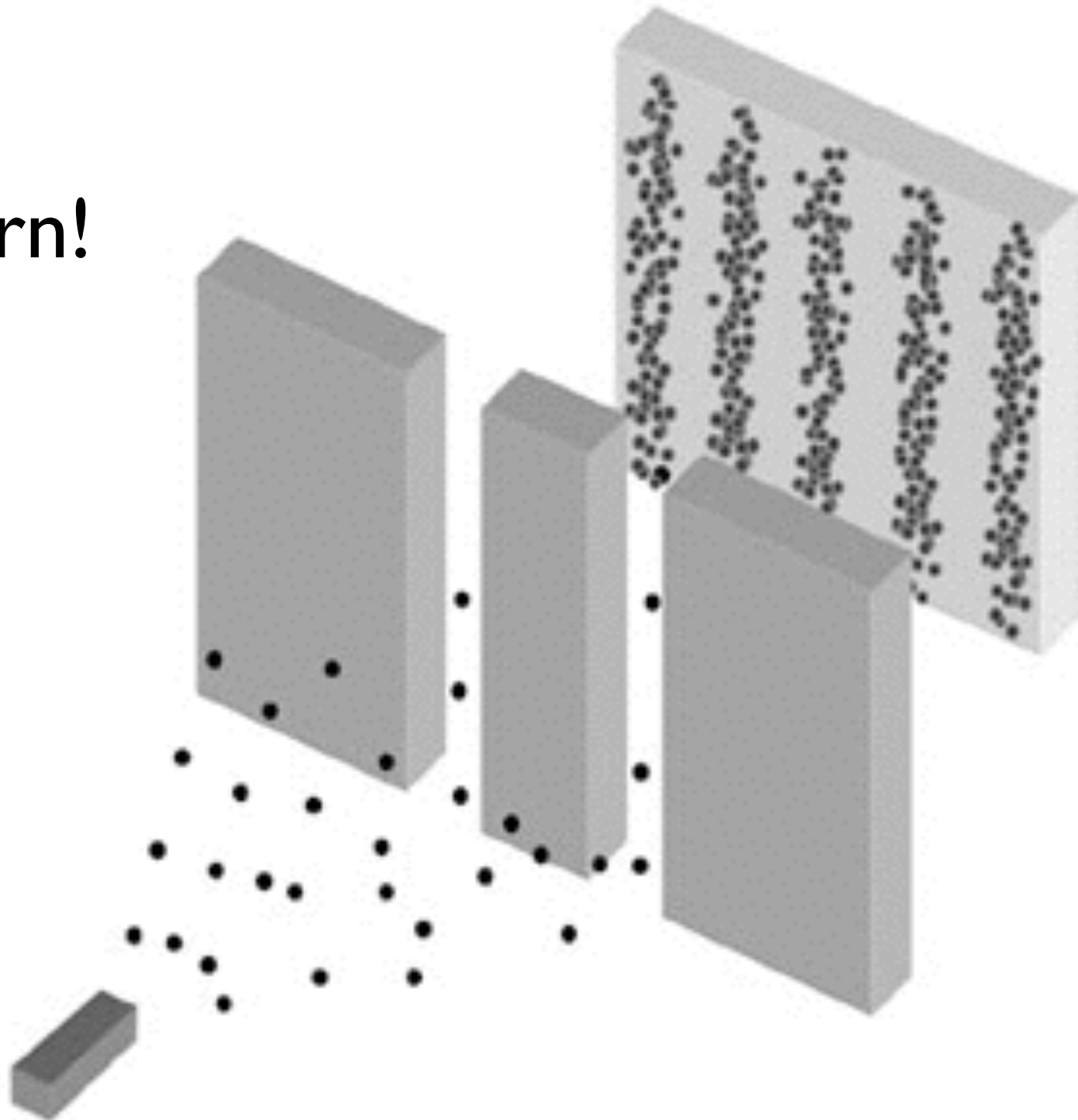
Interference pattern!

Waves of what?

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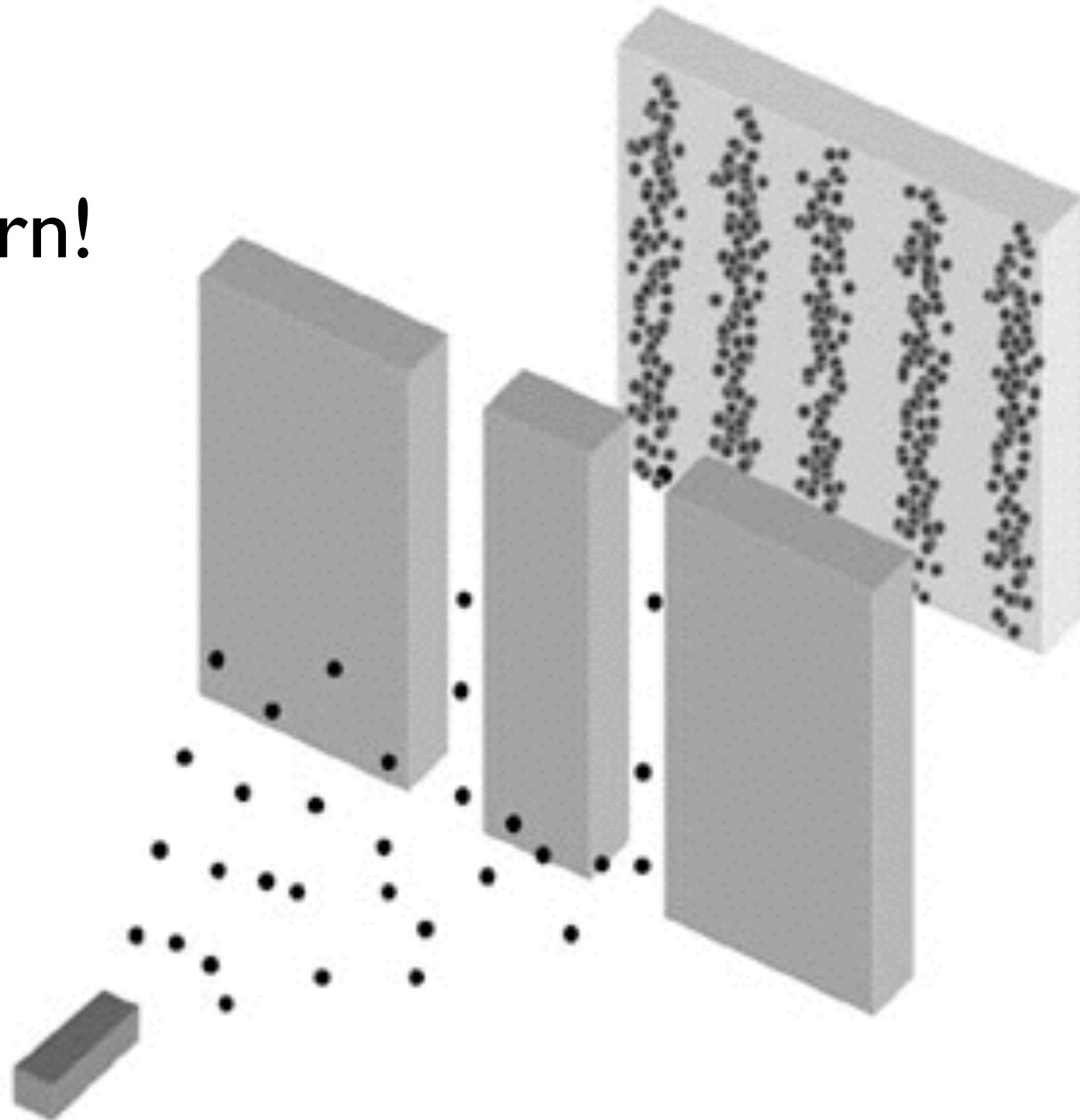
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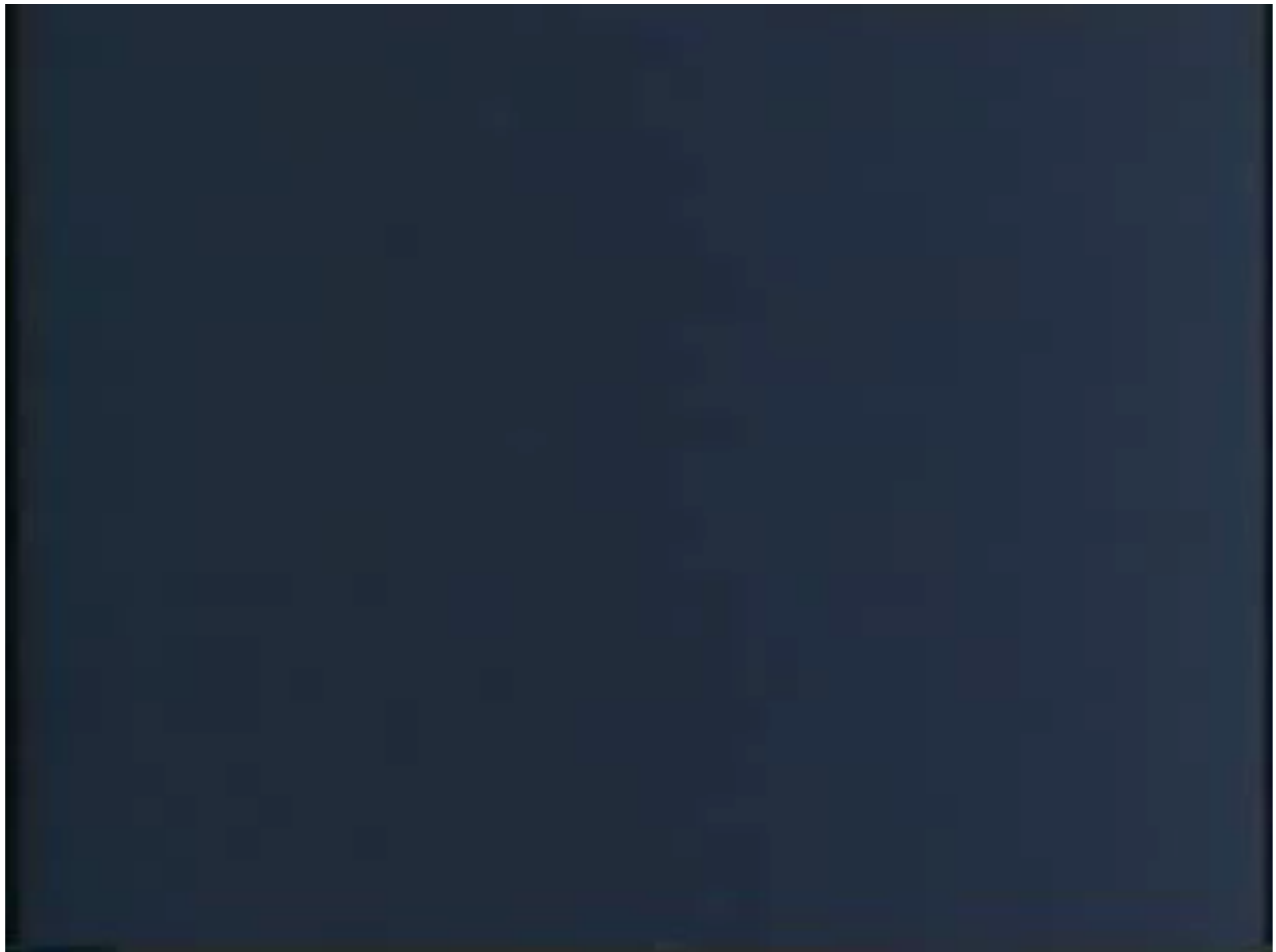
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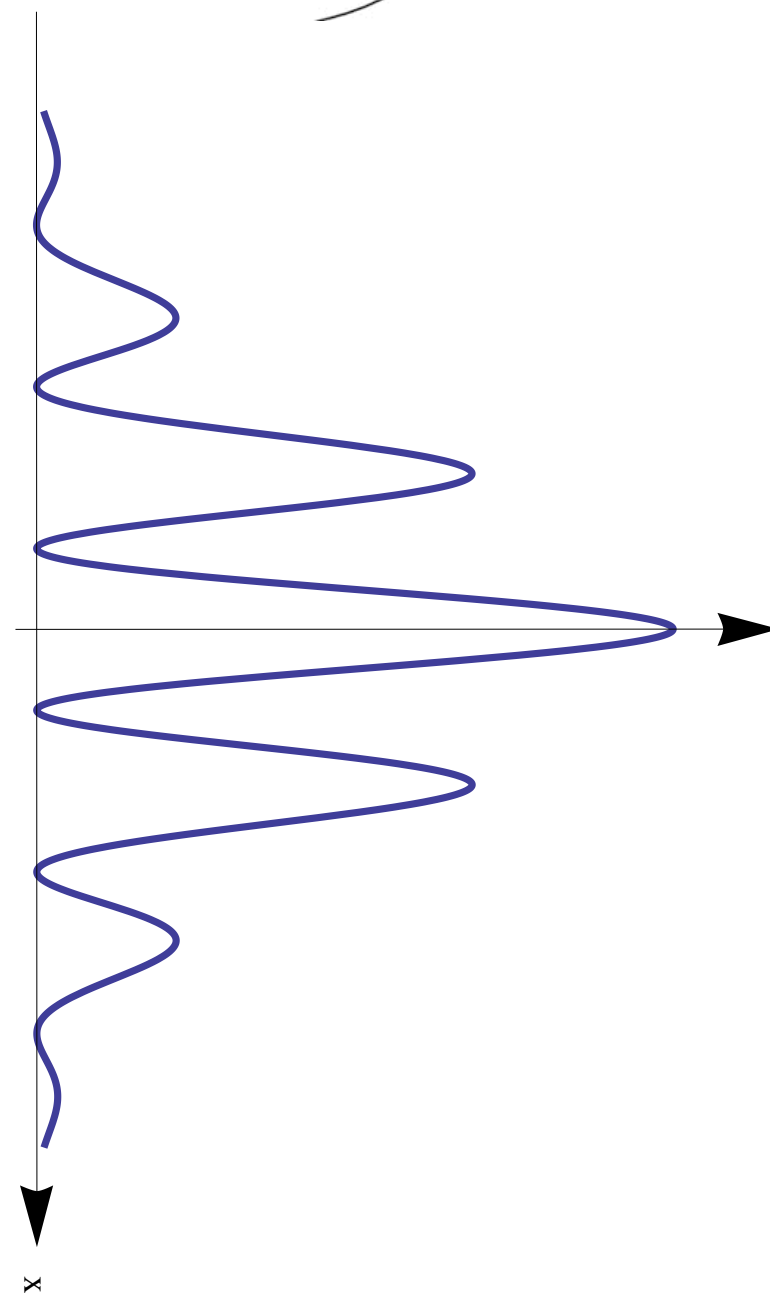
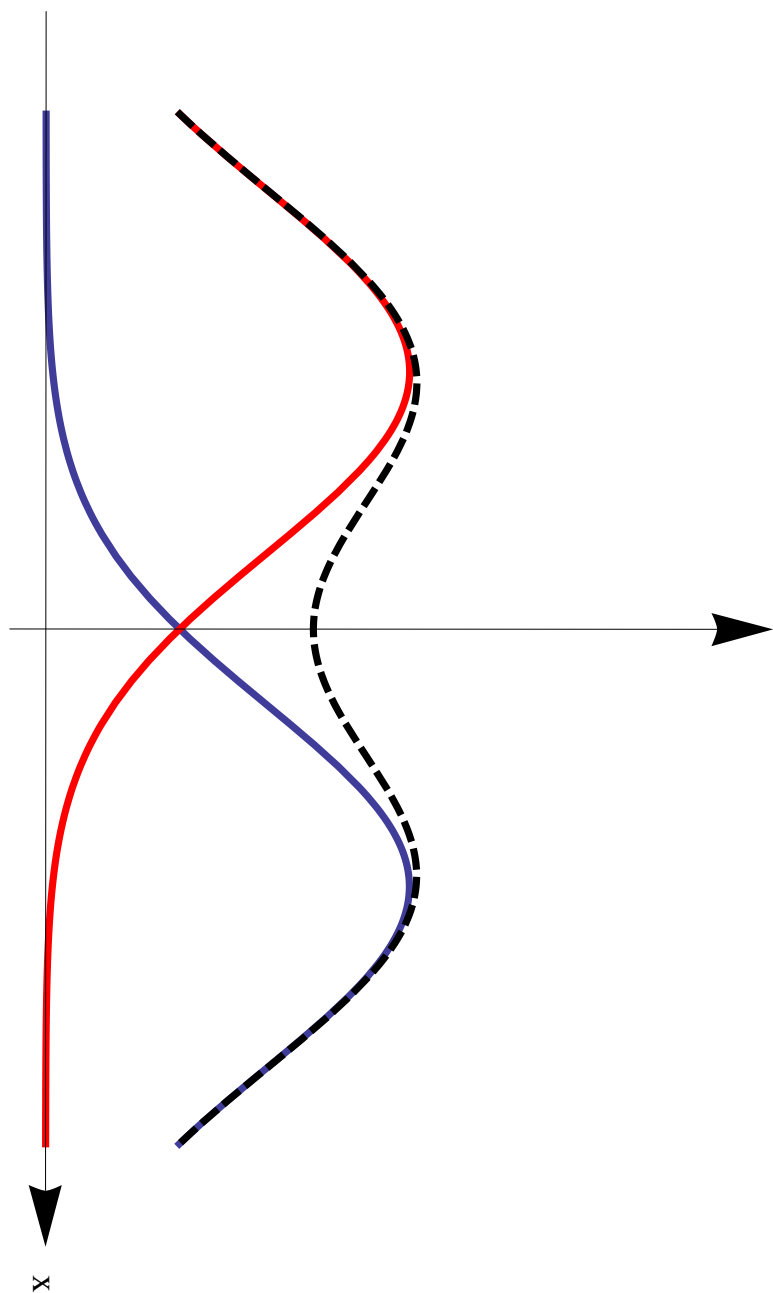
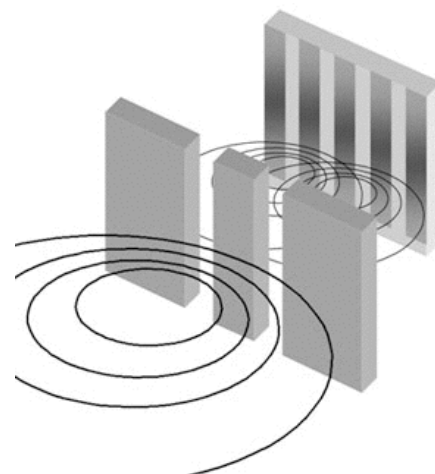
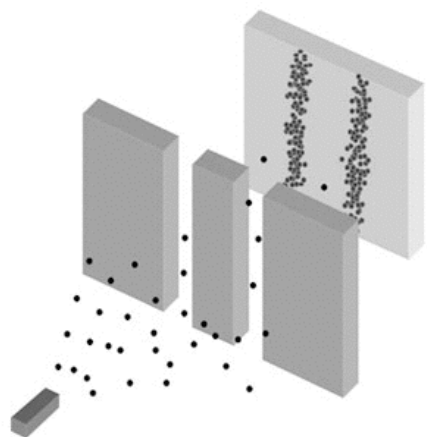
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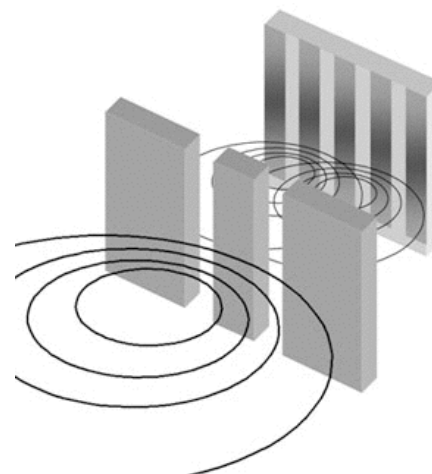
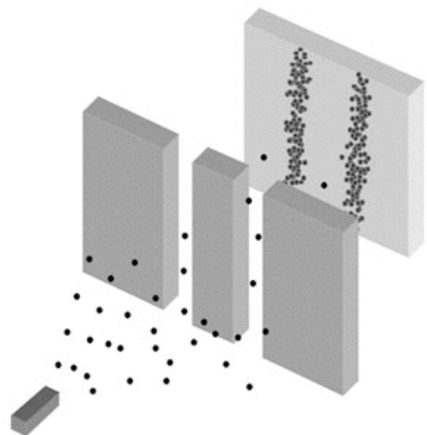




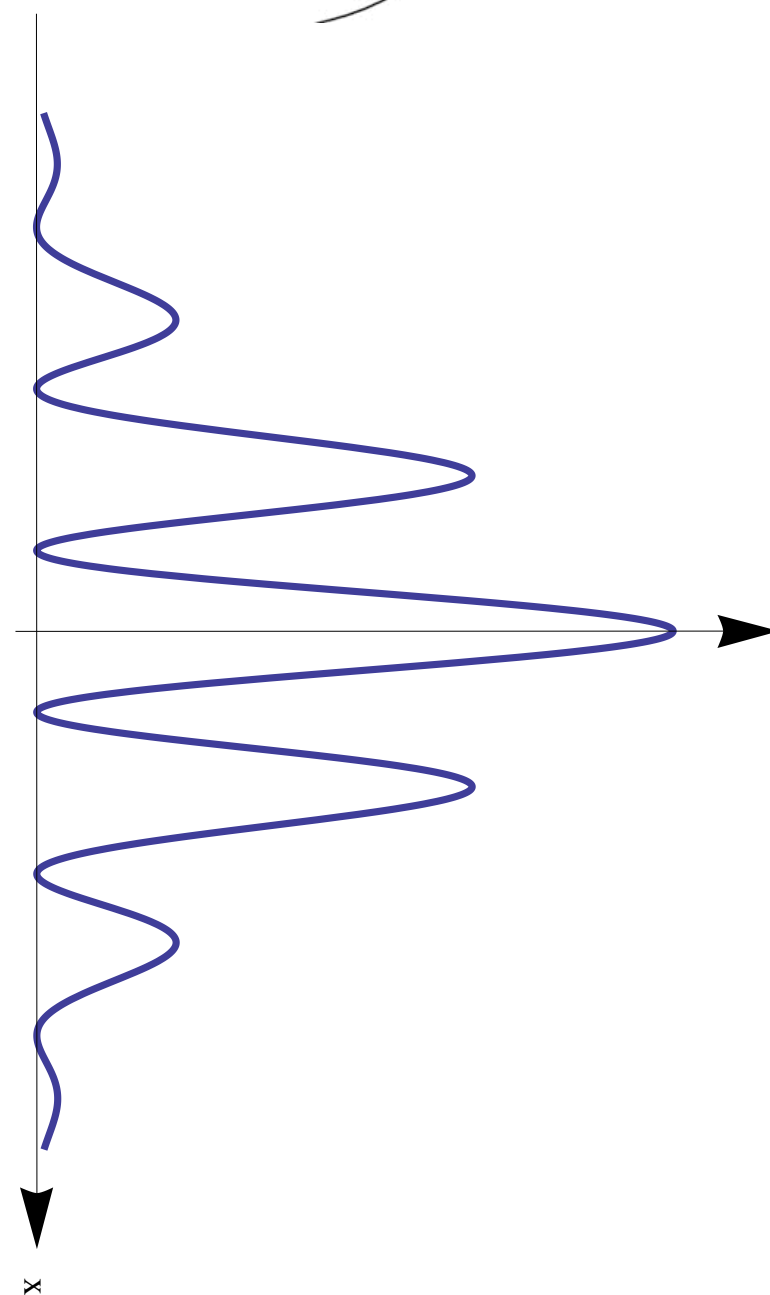
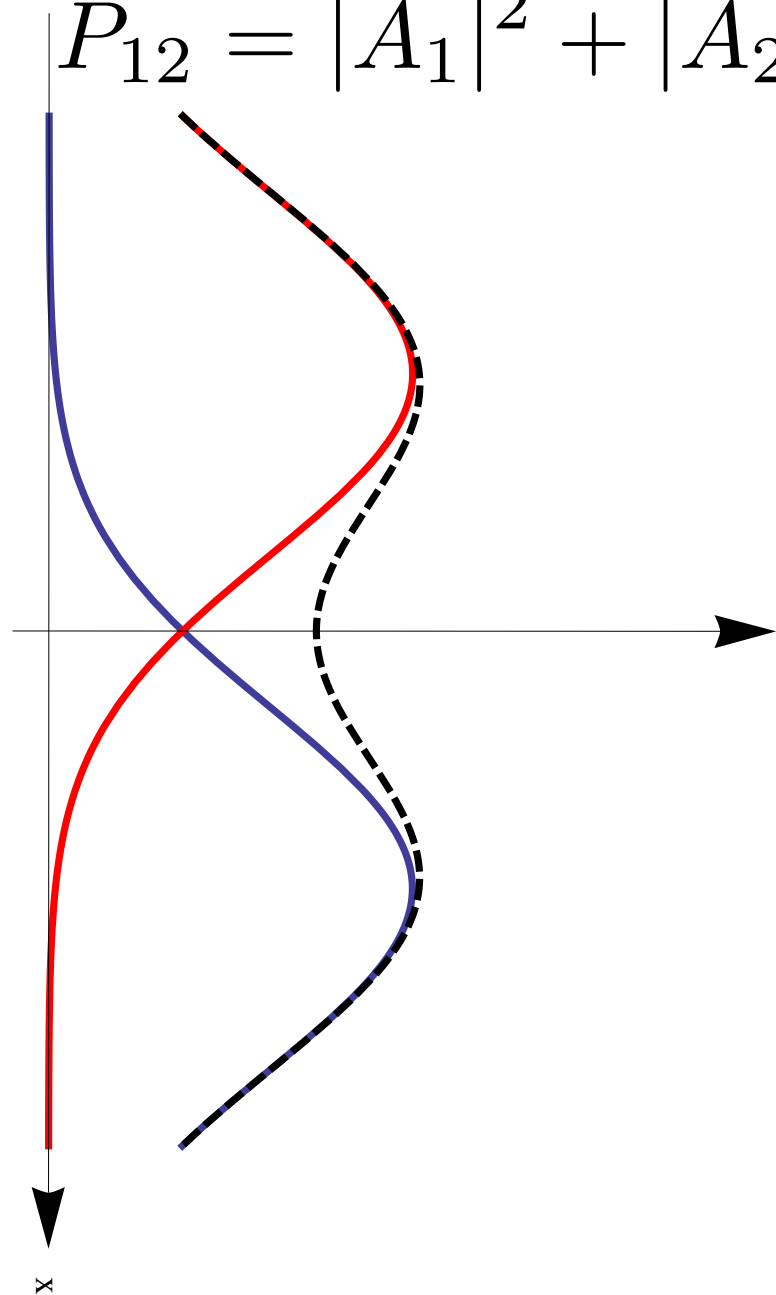


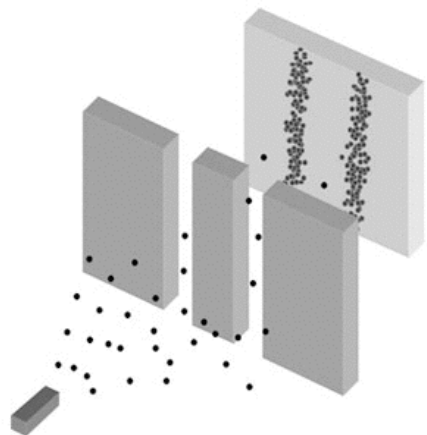




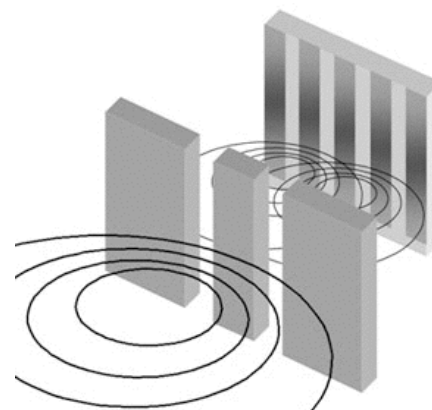
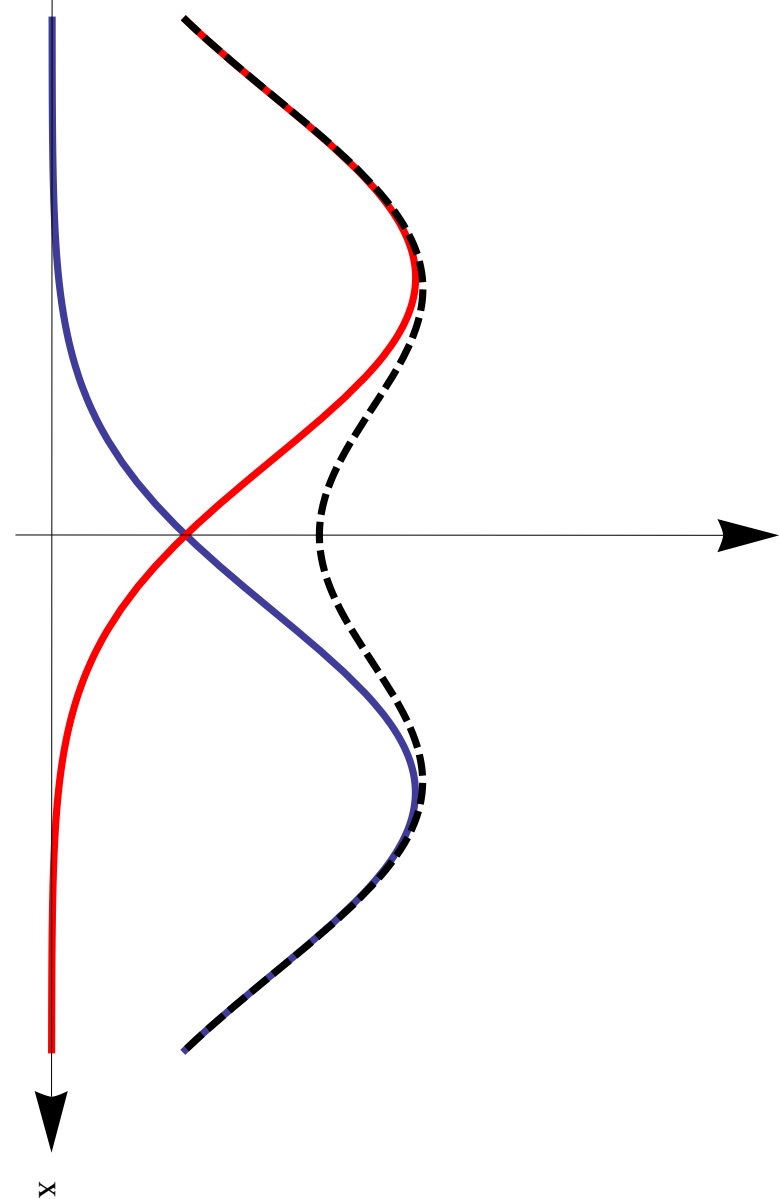


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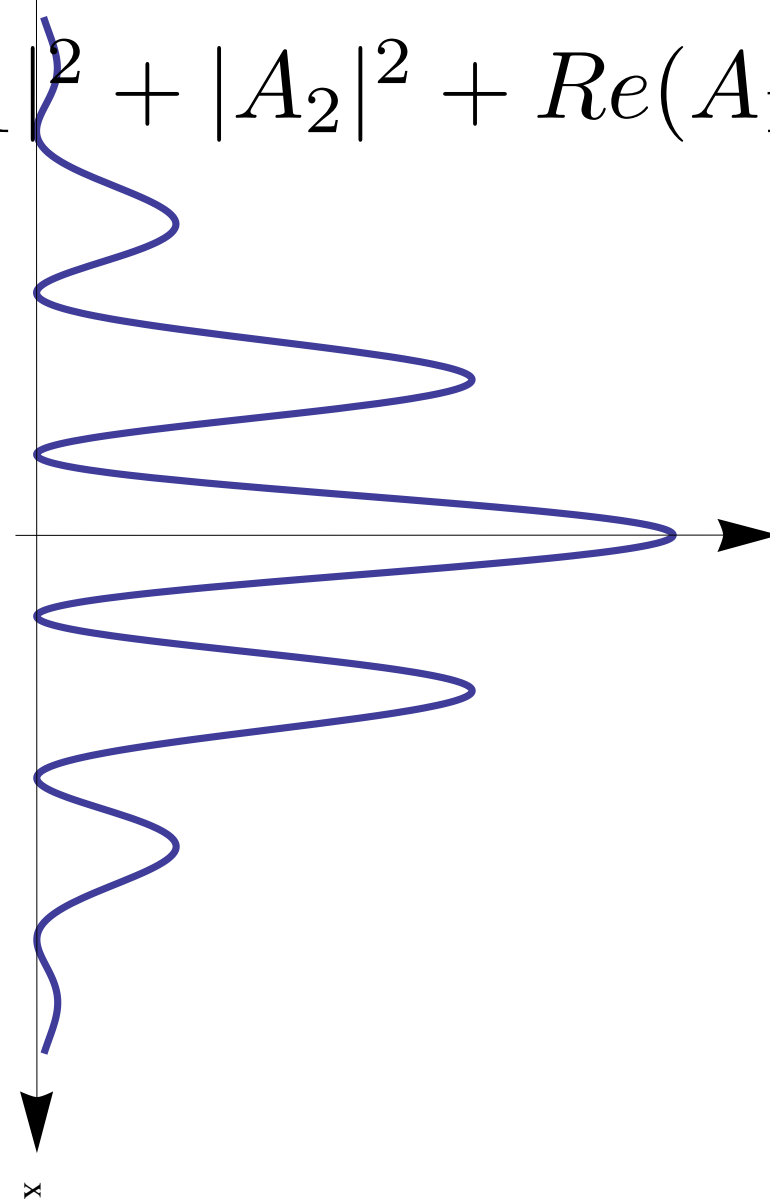




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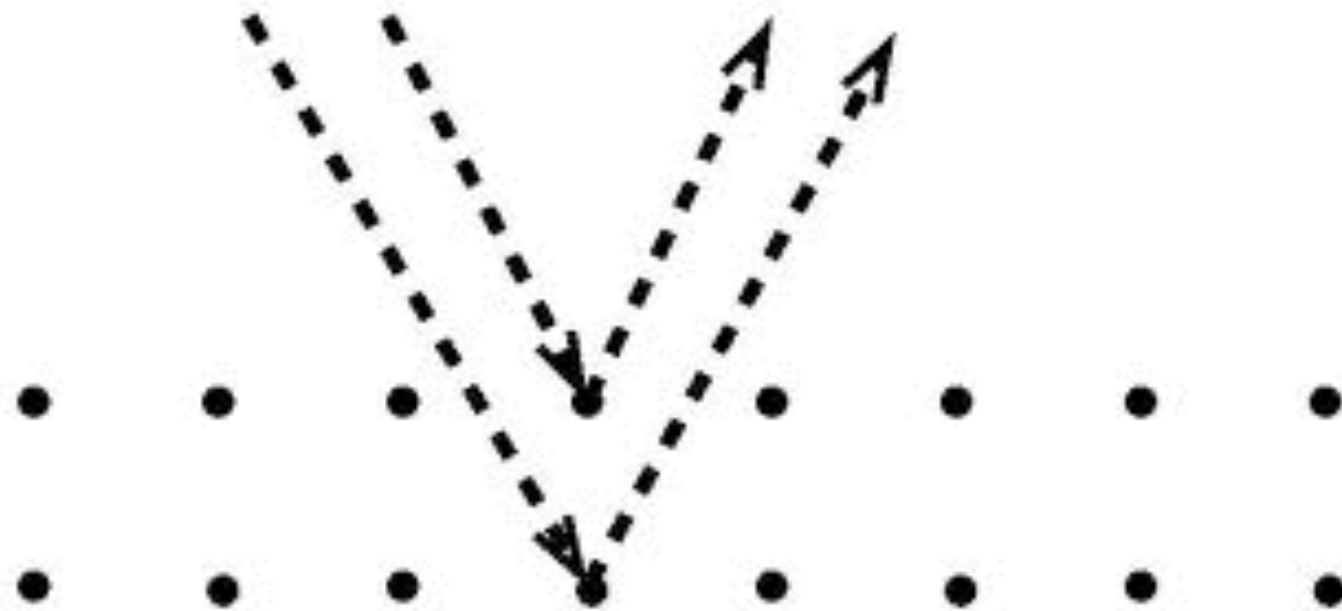
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# Gedanken vs real experiments

Interference occurs if wavelength  $\sim$  slit separation

de Broglie wavelength for an electron at 60 km/s is comparable to the atomic spacing in a metal





# The quantum mechanics wave function

$$\Psi(x, t)$$

The probability for a particle to be at a point (x,t) is

$$|\Psi(x, t)|^2$$

Quantum mechanics is a probabilistic theory

Evolution is deterministic but predictions are only statistical

# The Schrödinger equation



Erwin Schrödinger

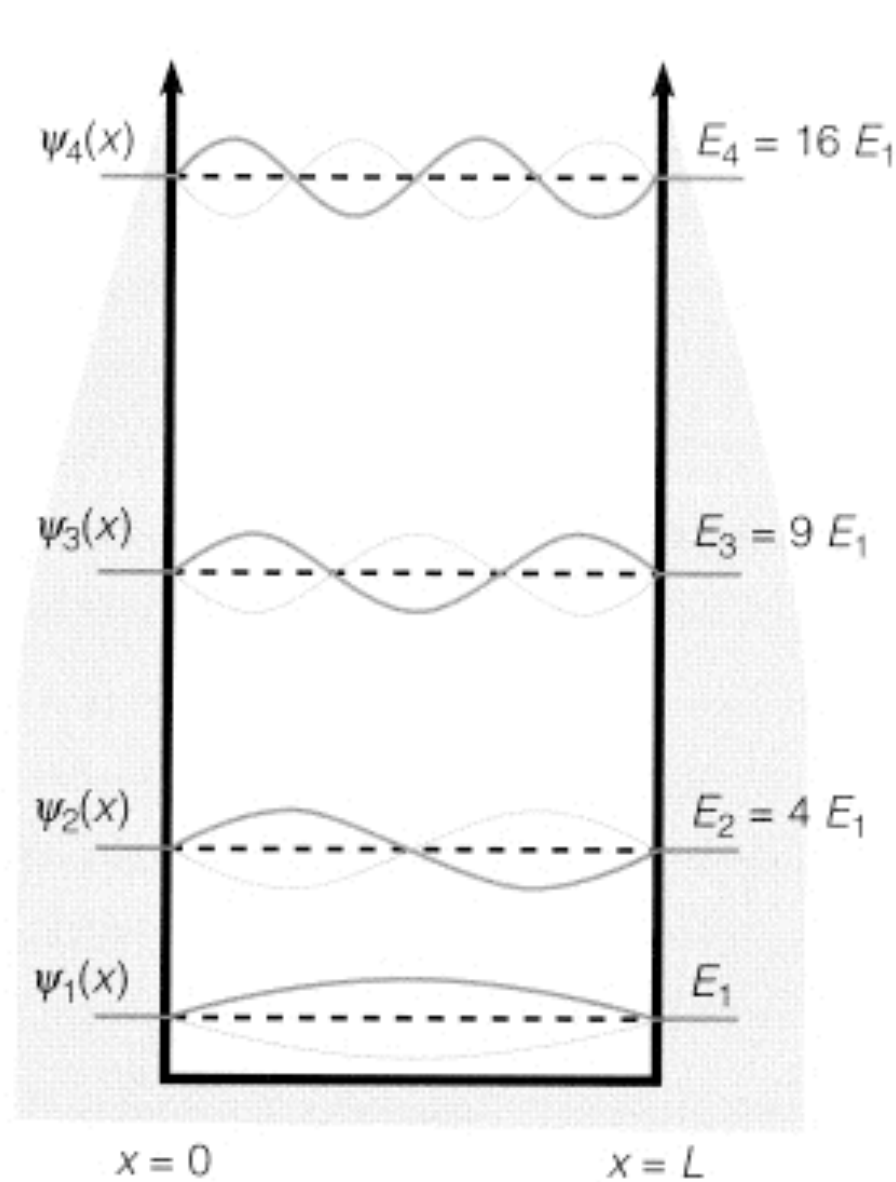
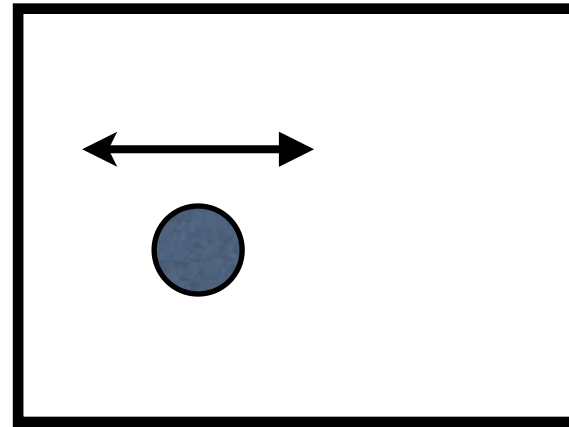


There is an equation that governs the evolution of the wavefunction

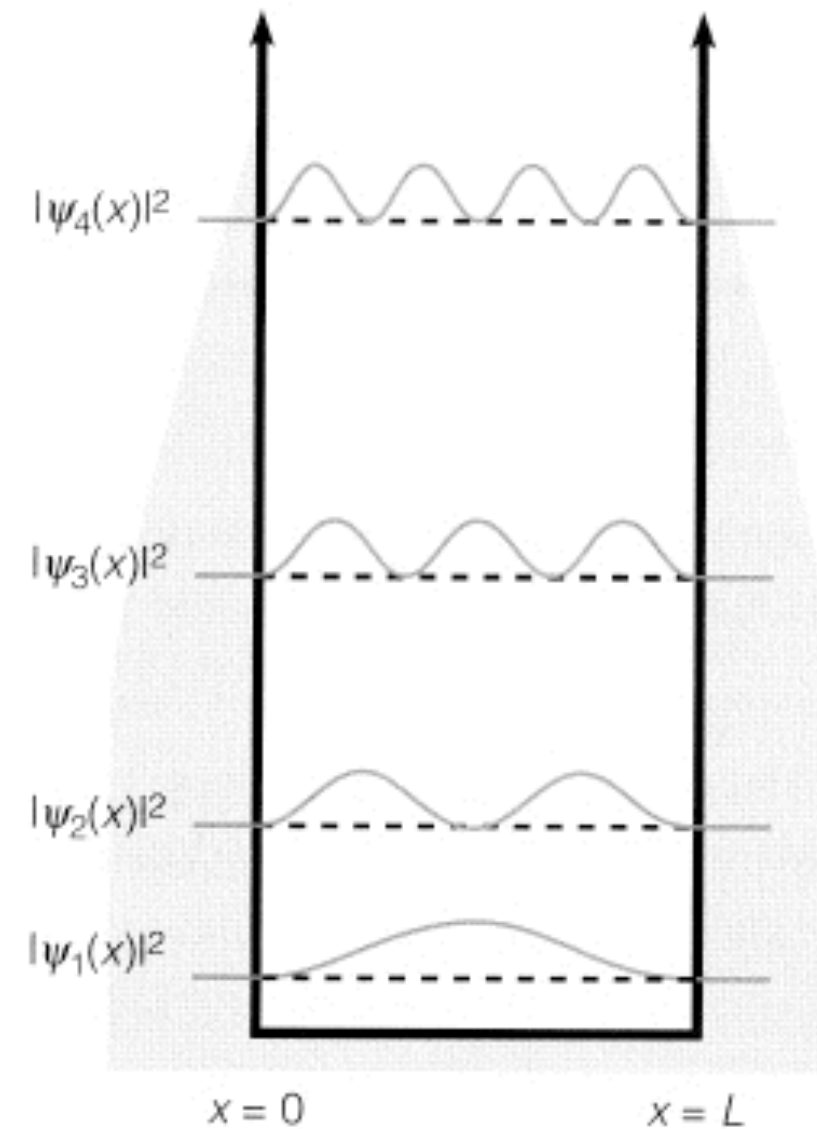
Can only ask questions like: what is the probability of seeing X?

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

# Particle trapped in a box



Infinite-well wave functions.



Infinite-well probability

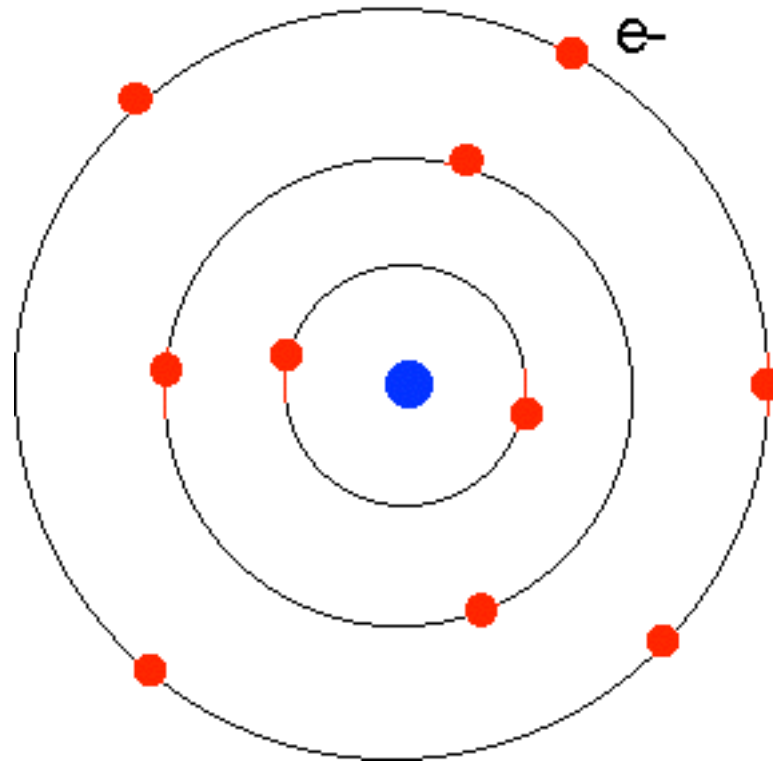
densities.

faster

In a classical world you wouldn't be here!



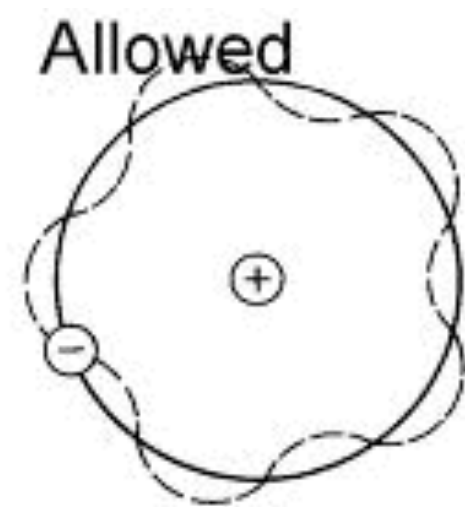
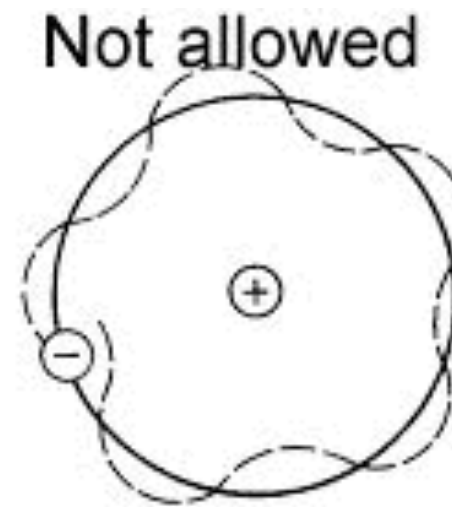
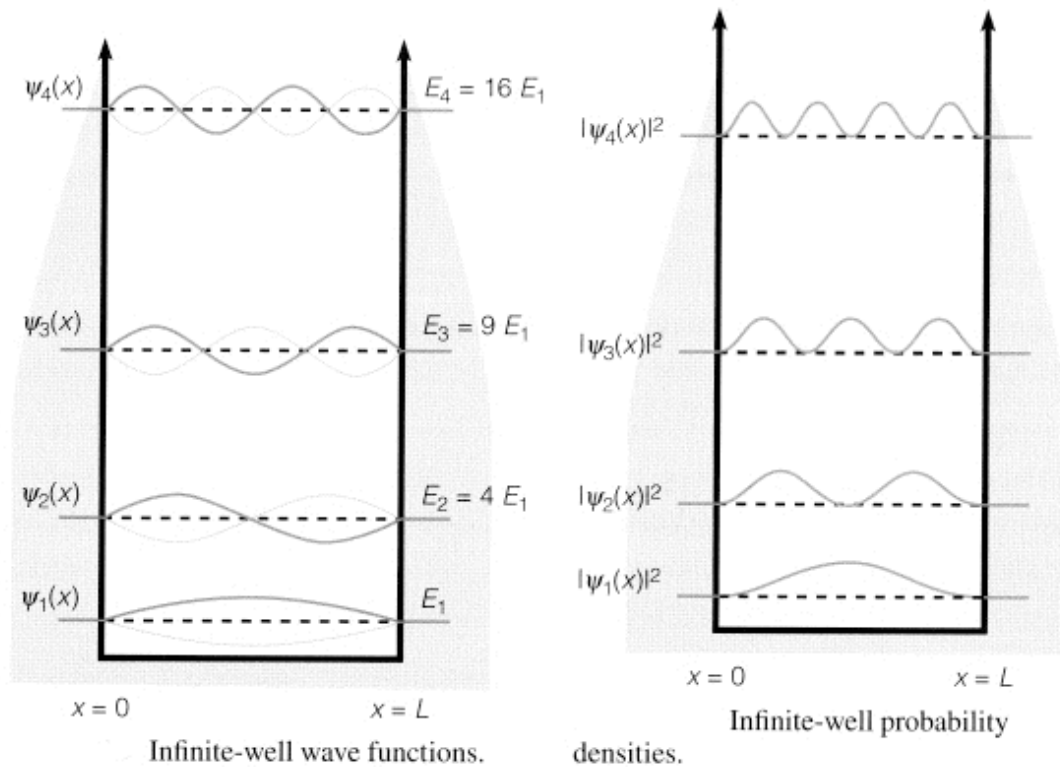
Neils Bohr 🏆



Unstable!

The Bohr atom, with quantum mechanics is stable

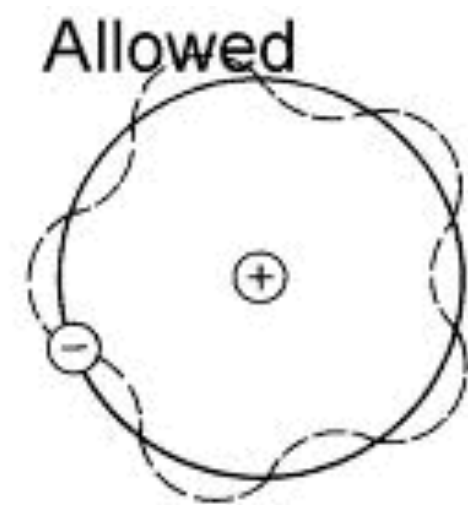
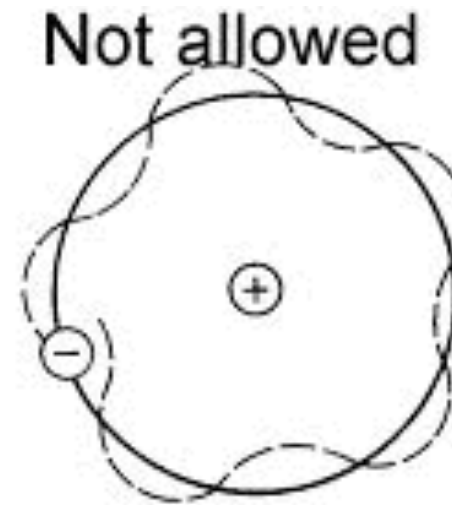
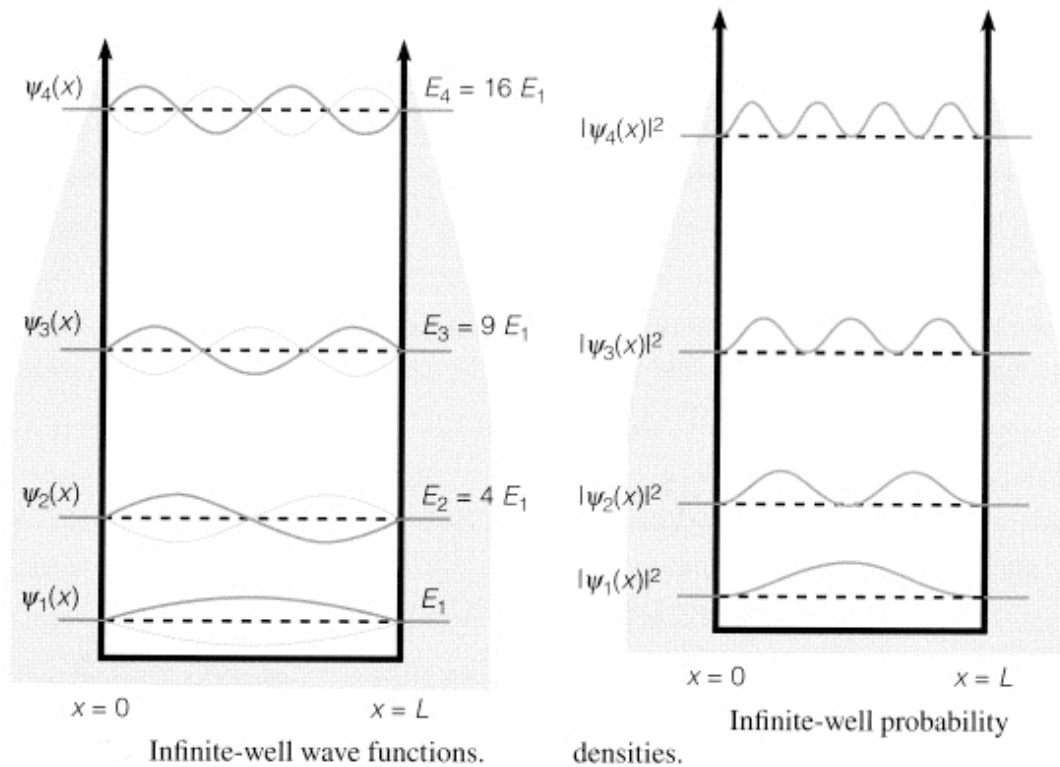
# Electrons as standing waves



Angular momentum is quantized

Lowest orbital has  $L=1$ , no where to decay to

# Electrons as standing waves



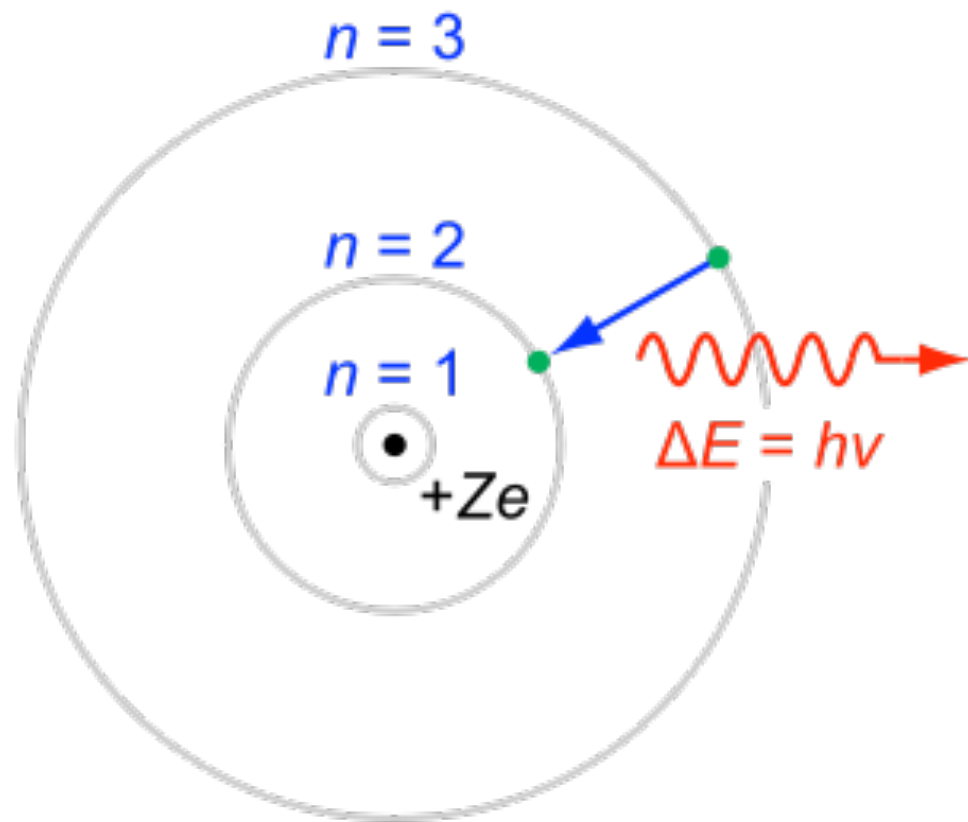
Angular momentum is quantized

Lowest orbital has  $L=1$ , no where to decay to

Atoms are stable



# The Bohr Atom



Light emitted/absorbed at fixed frequencies

Continuous Spectrum



Emission Lines



Absorption Lines



$$\nu \sim \frac{1}{n_1^2} - \frac{1}{n_2^2}$$

# Heisenberg Uncertainty relation



Werner Heisenberg



$$\Delta p > \frac{\hbar}{a}$$

“The more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa.”

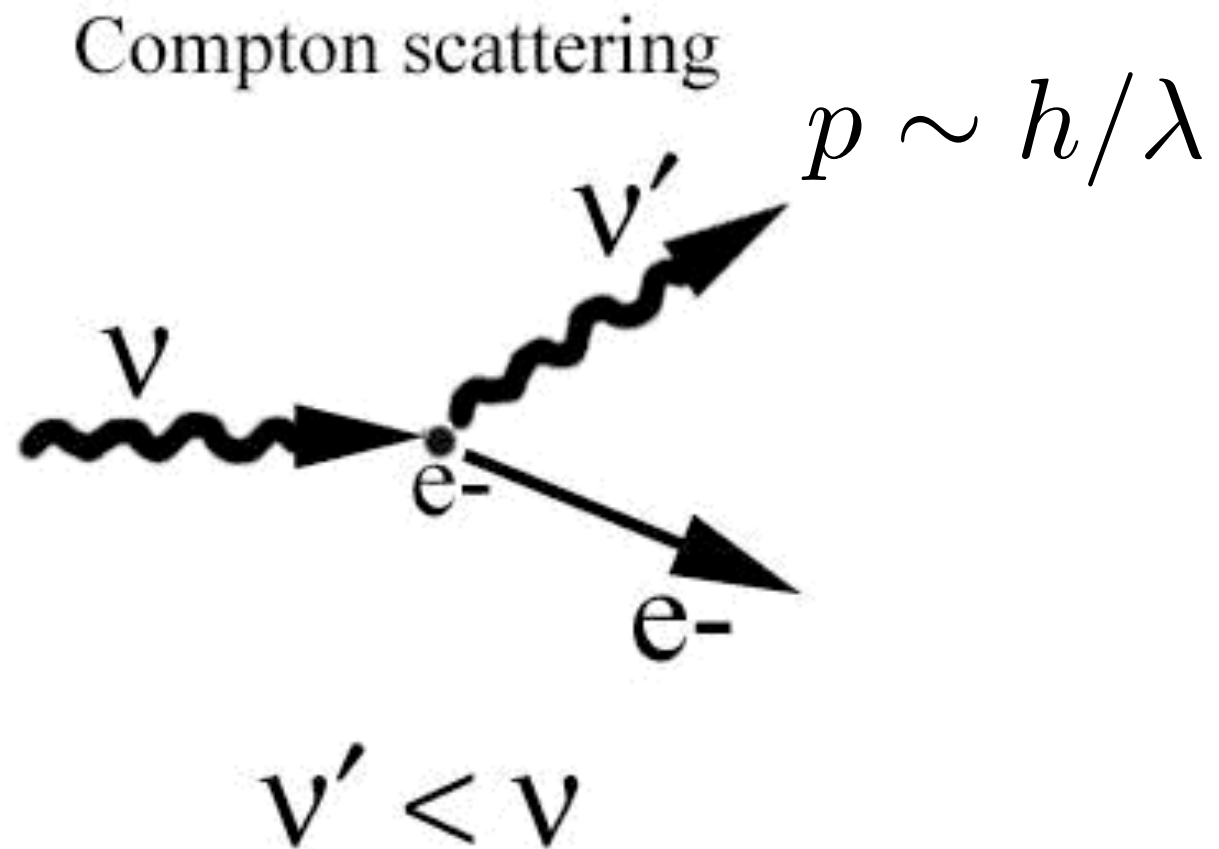
$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

The act of measurement disturbs the system

# Measuring a particle's position

Use a “microscope” - shine light on an electron

Shorter wavelength means better precision  $\Delta x \sim \lambda$

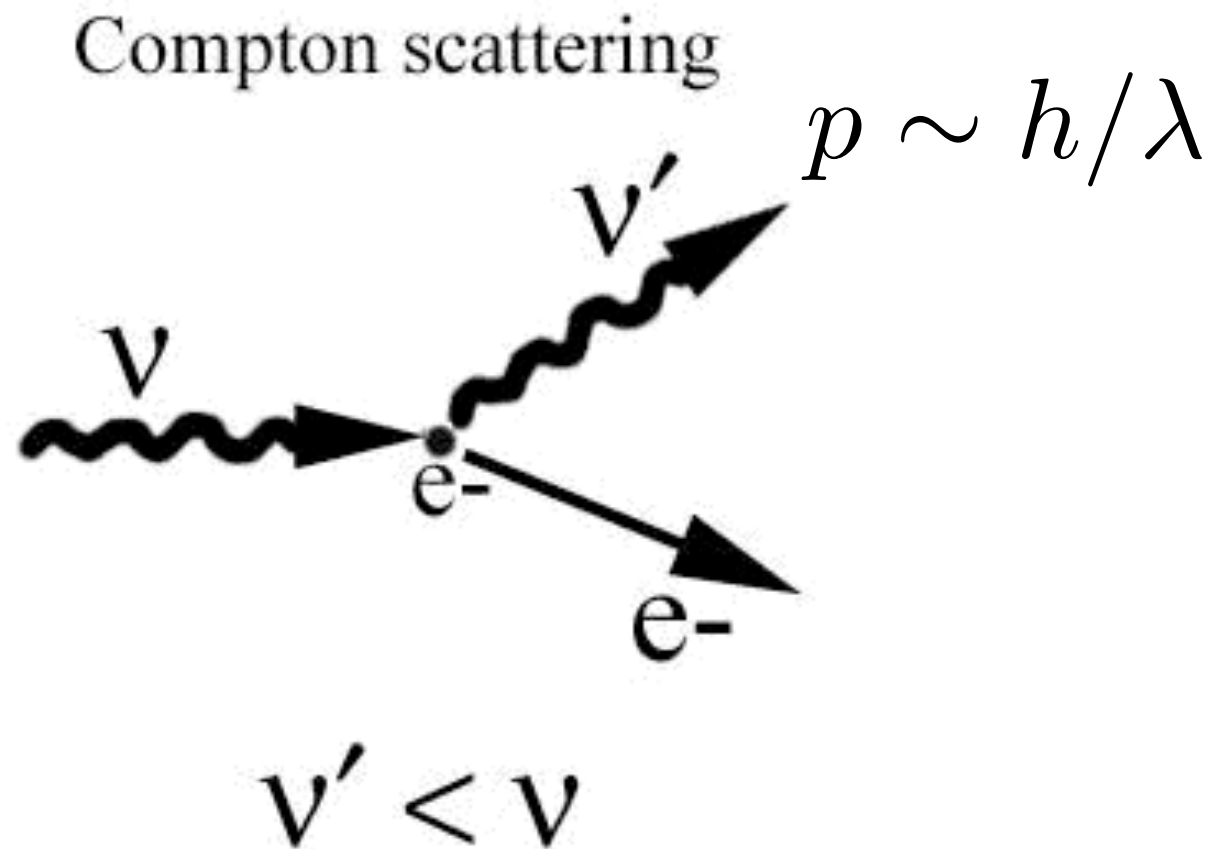


Electron is initially at rest  
 $e^-$  gains energy

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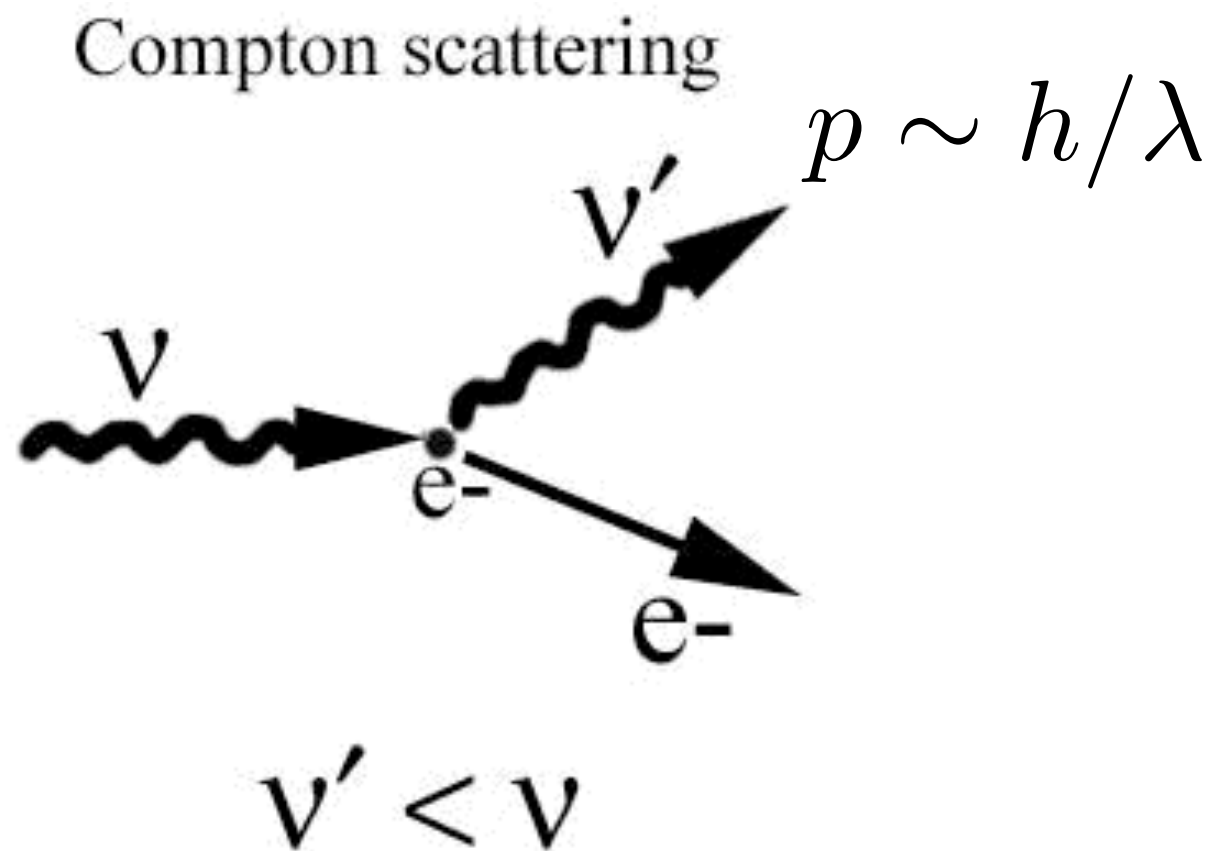
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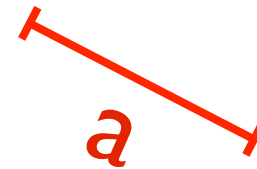
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Similar argument for  
Young's double slit

Monitor two slits, need  $\Delta x < \frac{a}{2}$

But then  $\Delta p > \frac{\hbar}{a}$

The fractional change in  
electrons momentum  
parallel to screen is  $\frac{\Delta p}{p} > \frac{\lambda}{a}$



Smears out  
interference pattern!

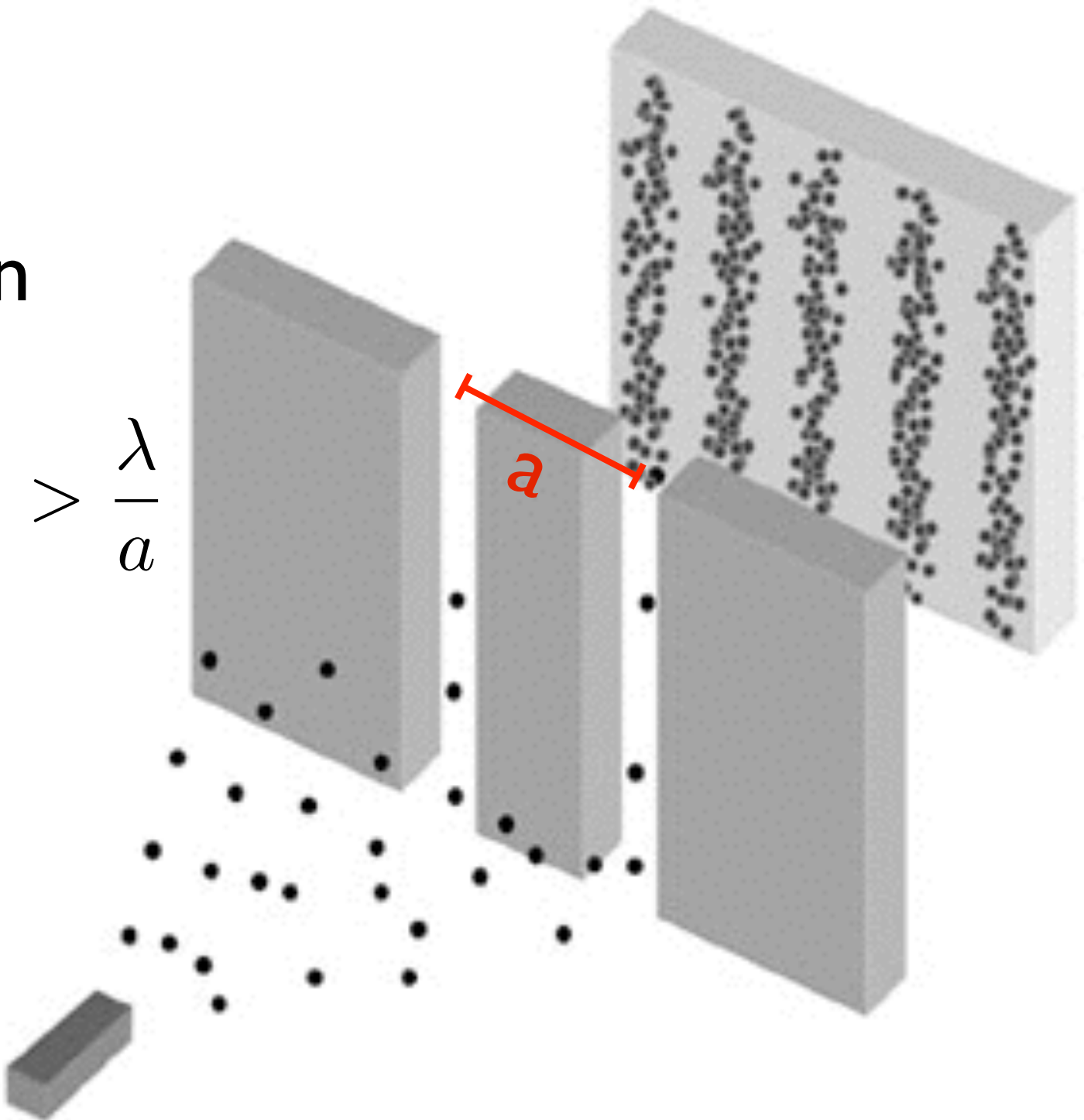


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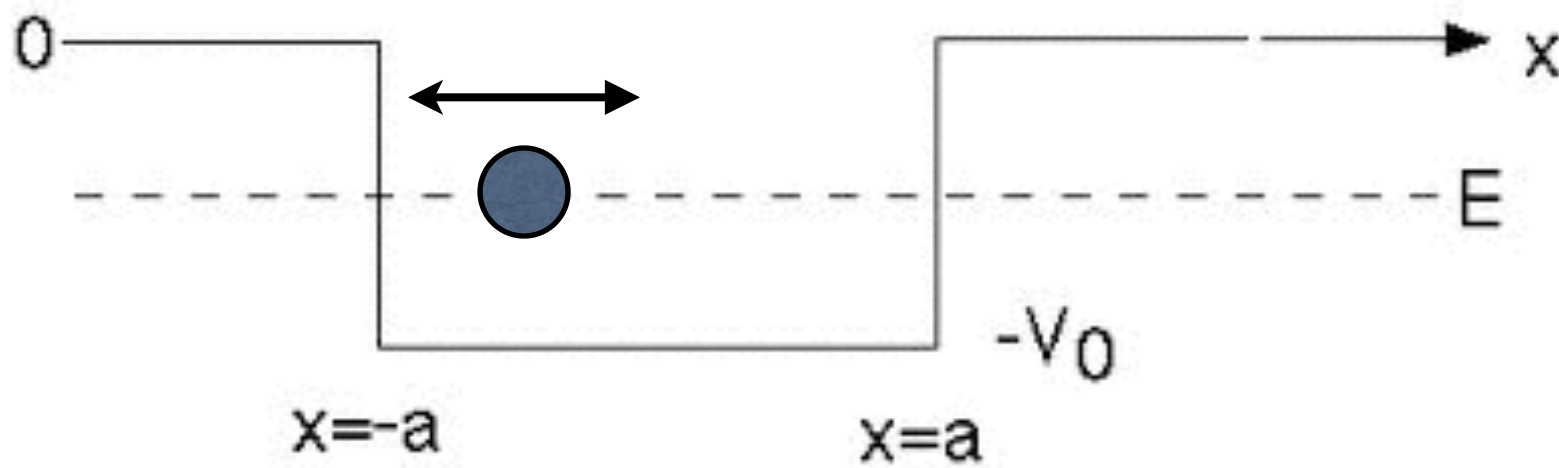
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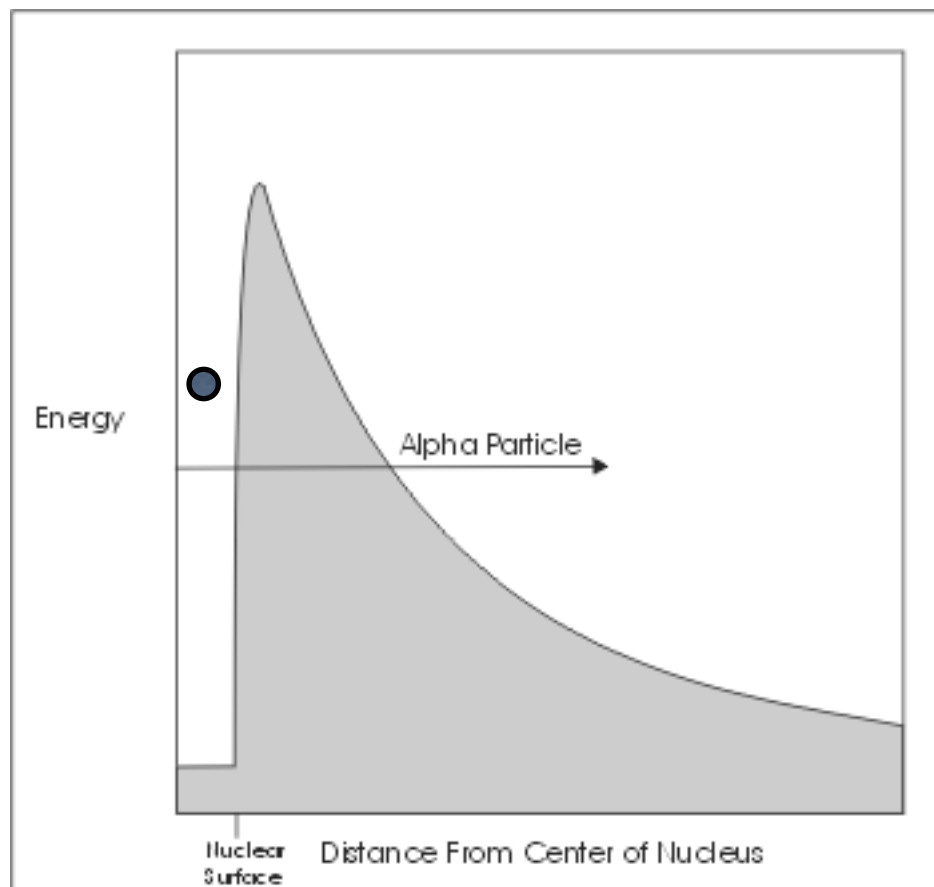


# Quantum tunneling

In QM the classically impossible is now just very unlikely

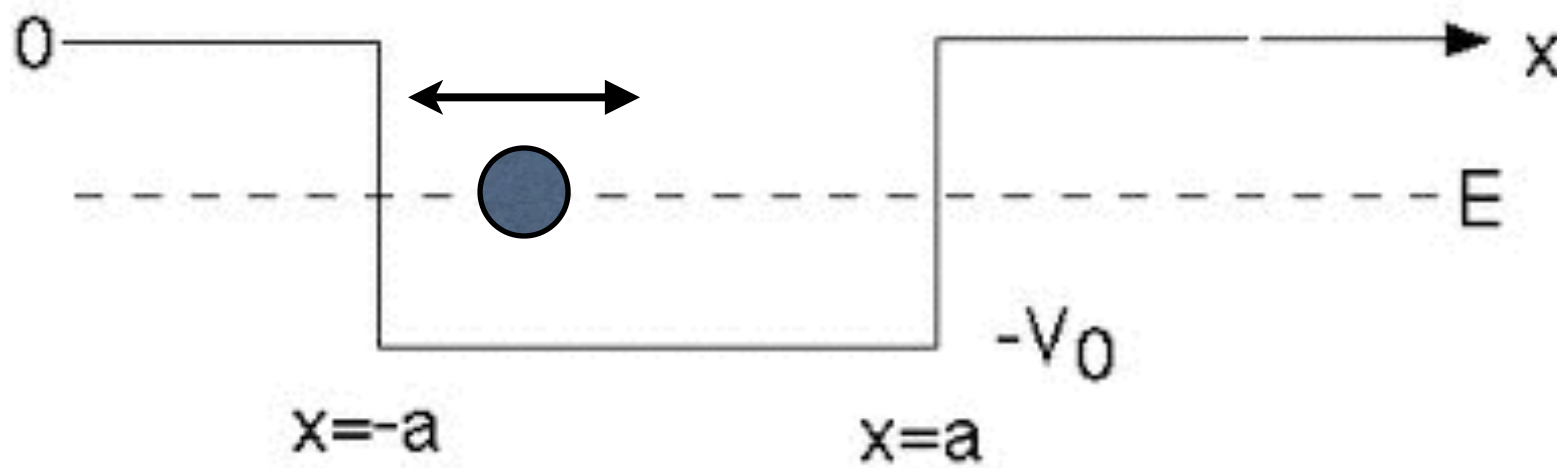


Uncertainty principle  
means it can be outside  
well

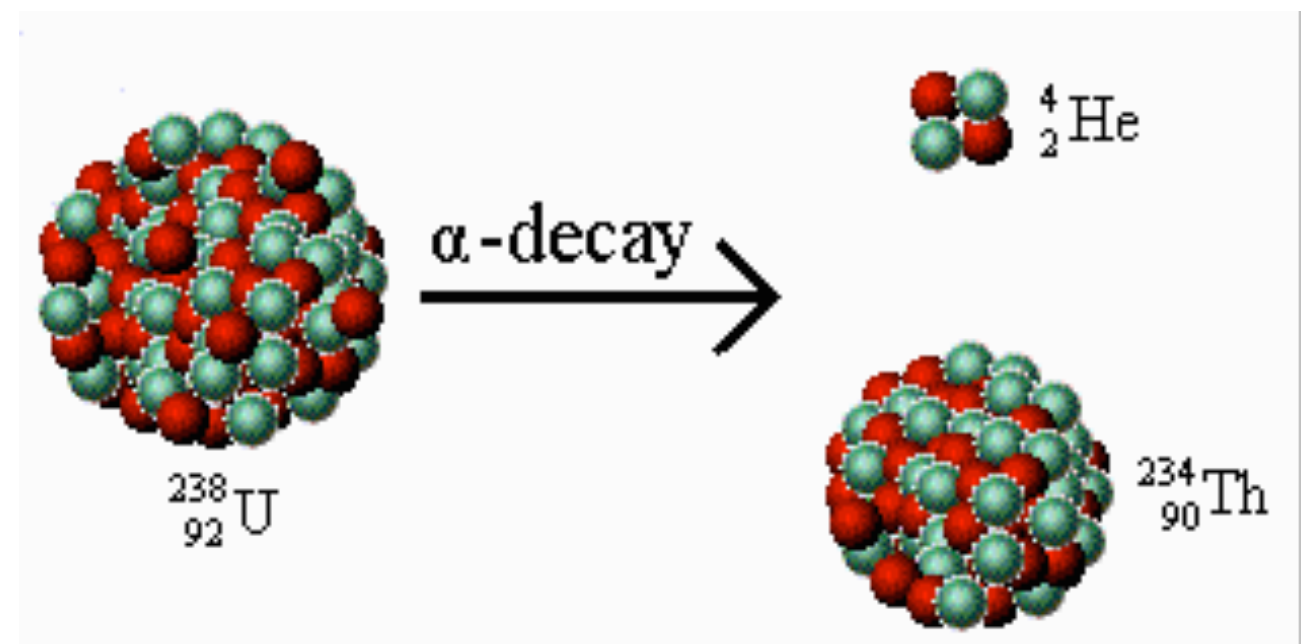
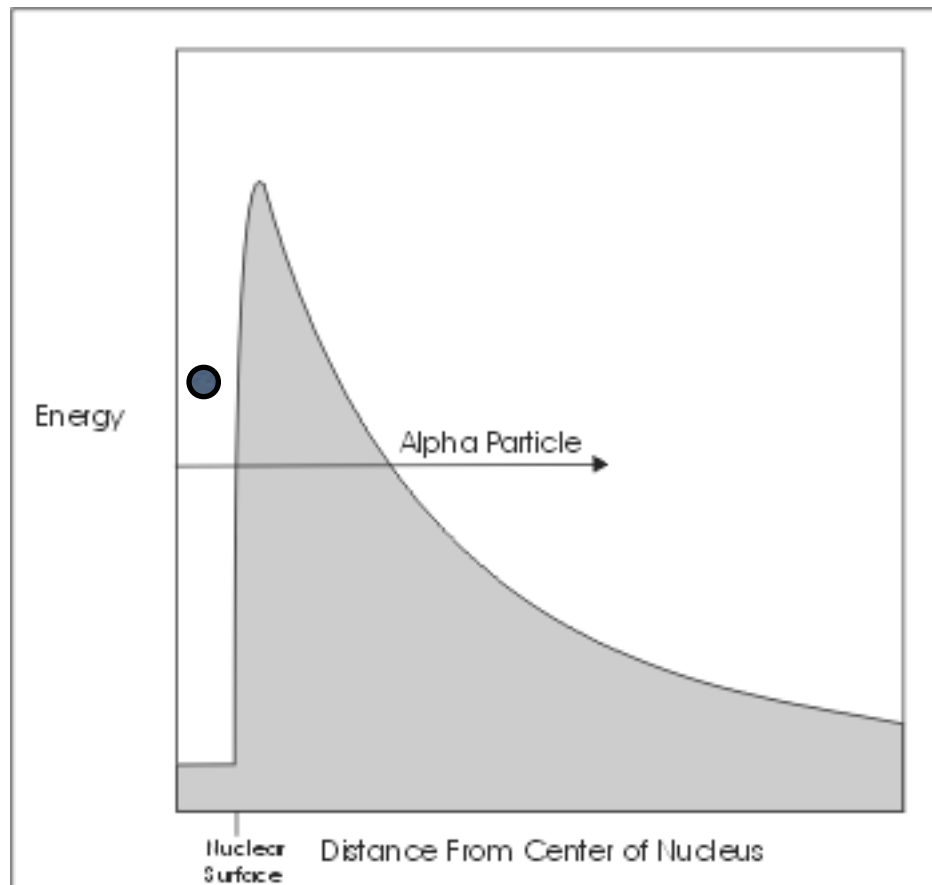


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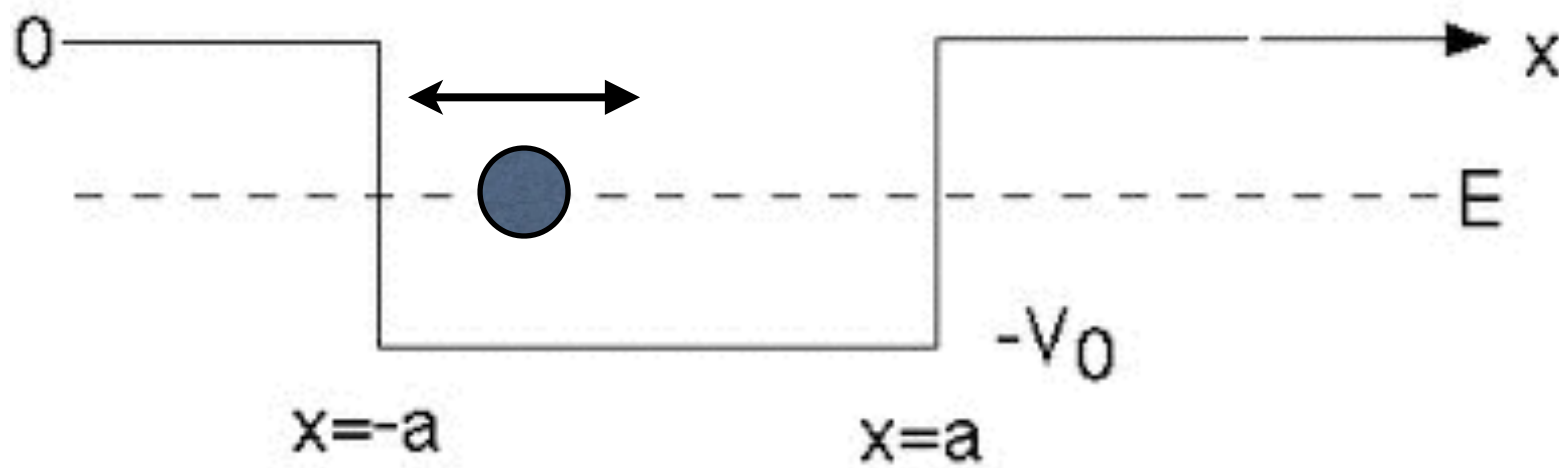


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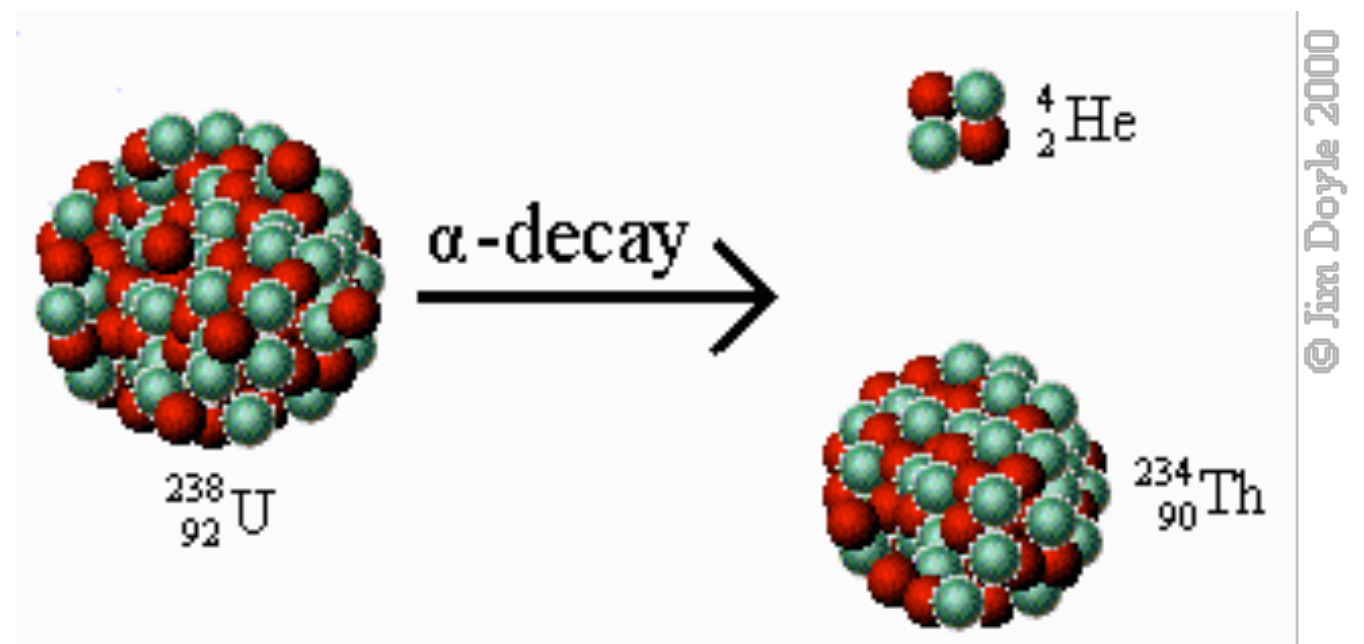
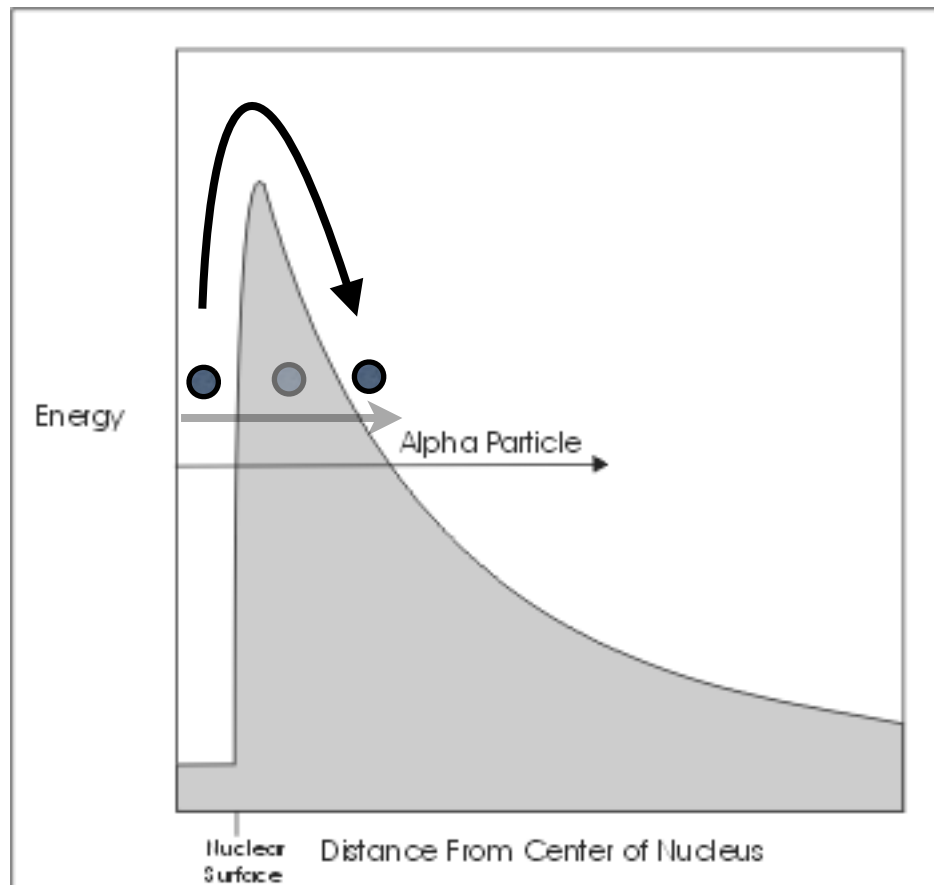


# Quantum tunneling

In QM the classically impossible is now just very unlikely



Uncertainty principle means it can be outside well



# Superpositions

$$\Psi = \sum_i \psi_i$$

Wavefunction is a superposition of all possibilities

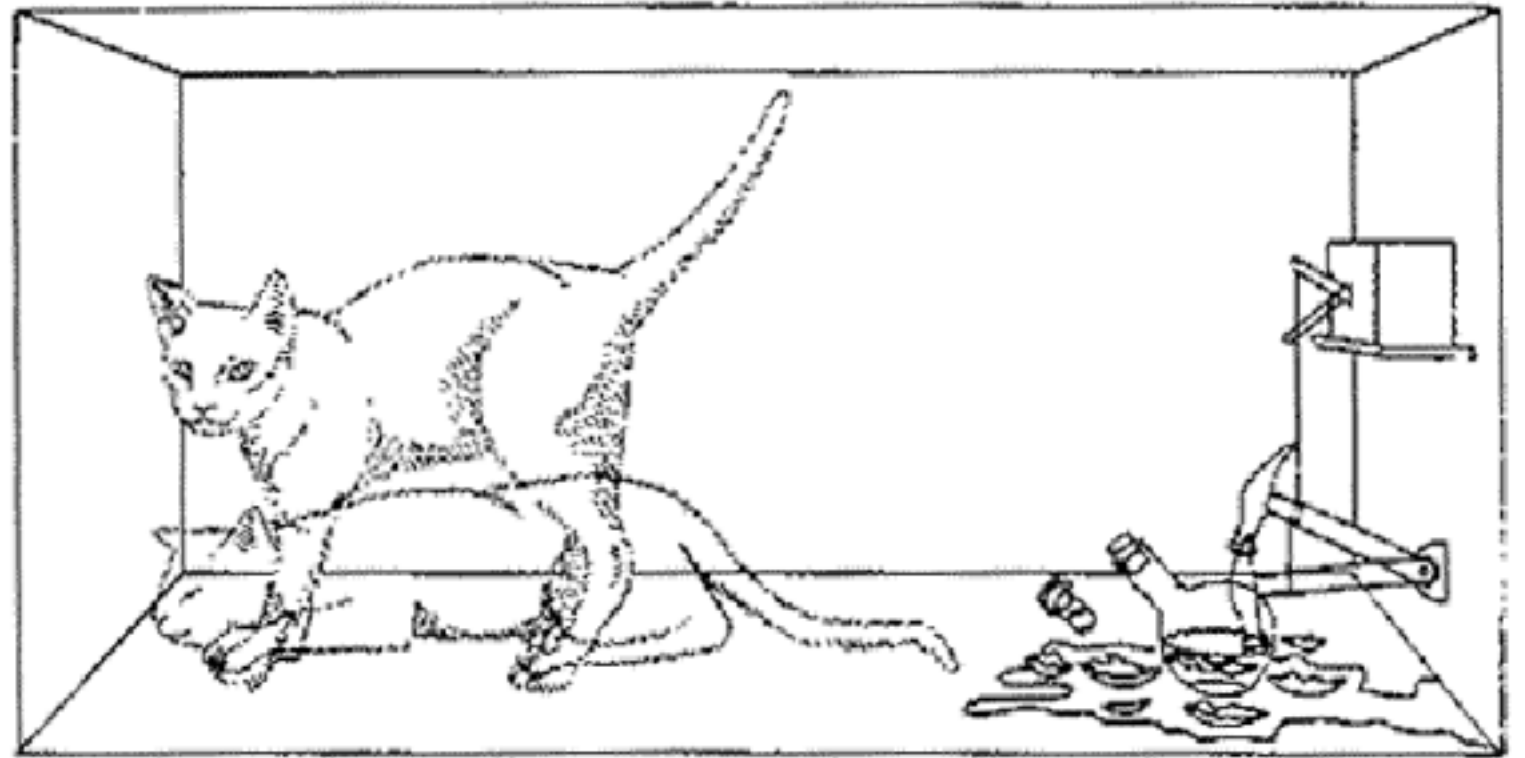
QM predicts outcomes if expt. is repeated many times

Measurement “collapses” the wavefunction

# Schrödinger's cat

## Copenhagen interpretation of QM

Cat is in a  
superposition of alive  
and dead until box is  
opened!

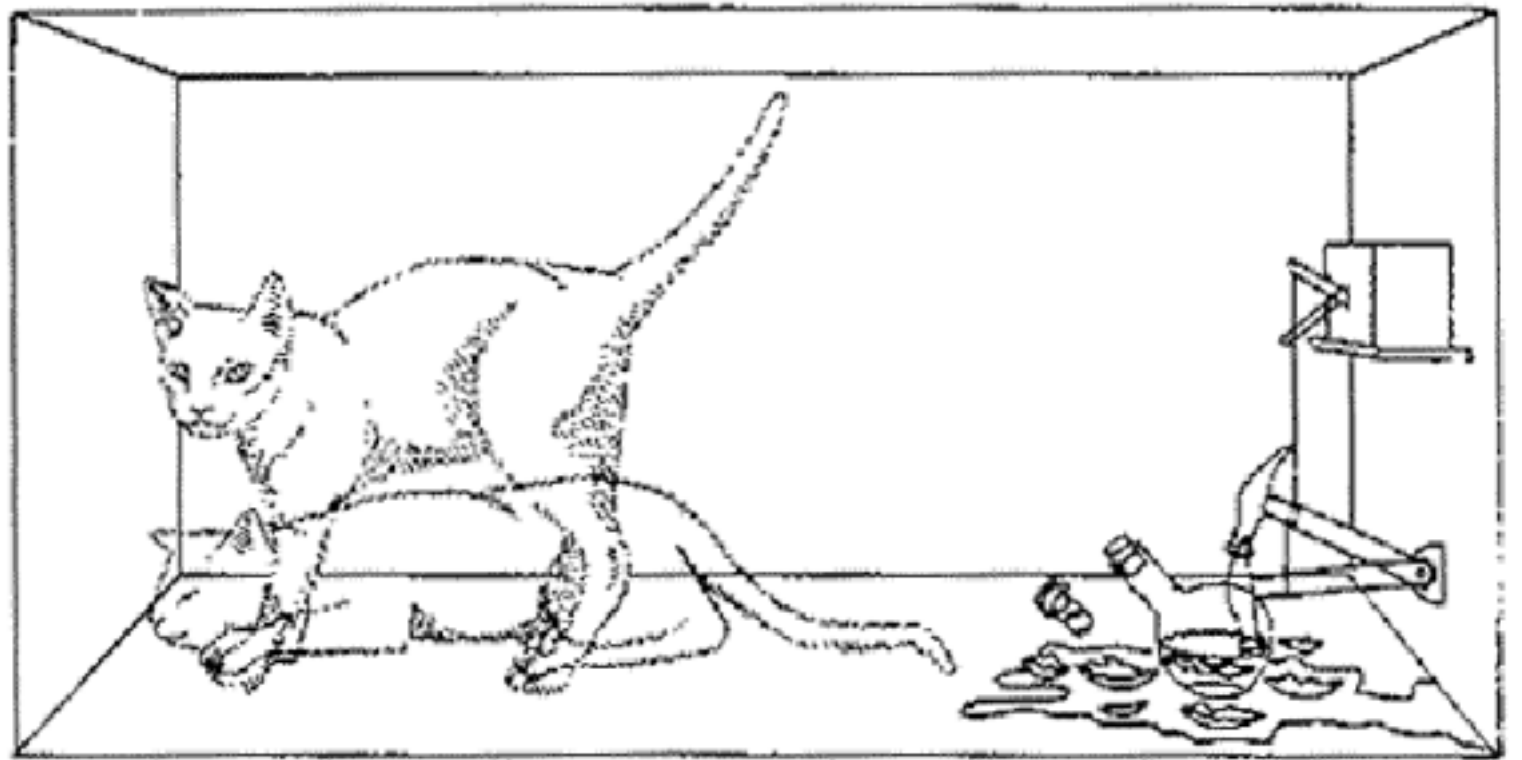




# Schrödinger's cat

## Copenhagen interpretation of QM

Cat is in a superposition of alive and dead until box is opened!



“Shut up and calculate”  
--David Mermin

# Applications of Quantum Mechanics

The birth of QM in the early 1900's lead to a profound change in our understanding of nature at short distances

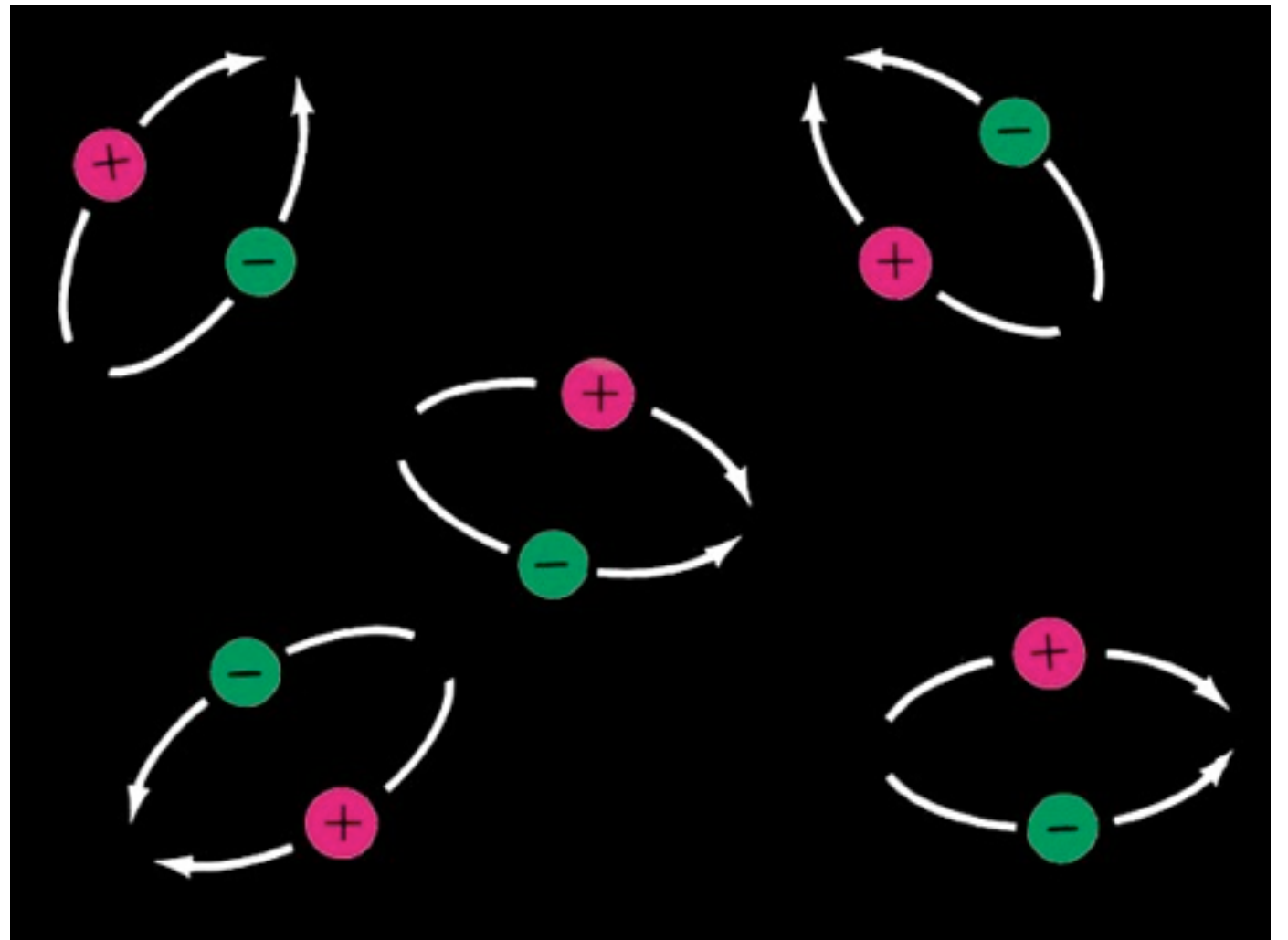
Without this understanding we would not have transistors  
(and all semi-conductors), lasers, medical imaging  
technology, superconductors,...Fermilab!

Quantum Mechanics is not the final story.....

# The loss of empty space



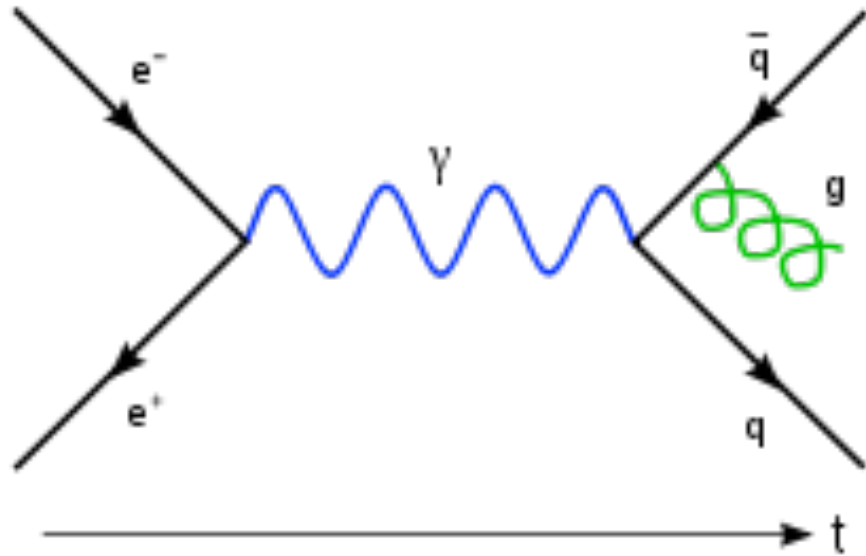
Paul A.M. Dirac



Dirac combined relativity and QM

Predicted antiparticles

# Quantum Electrodynamics (QED)



Feynman diagrams

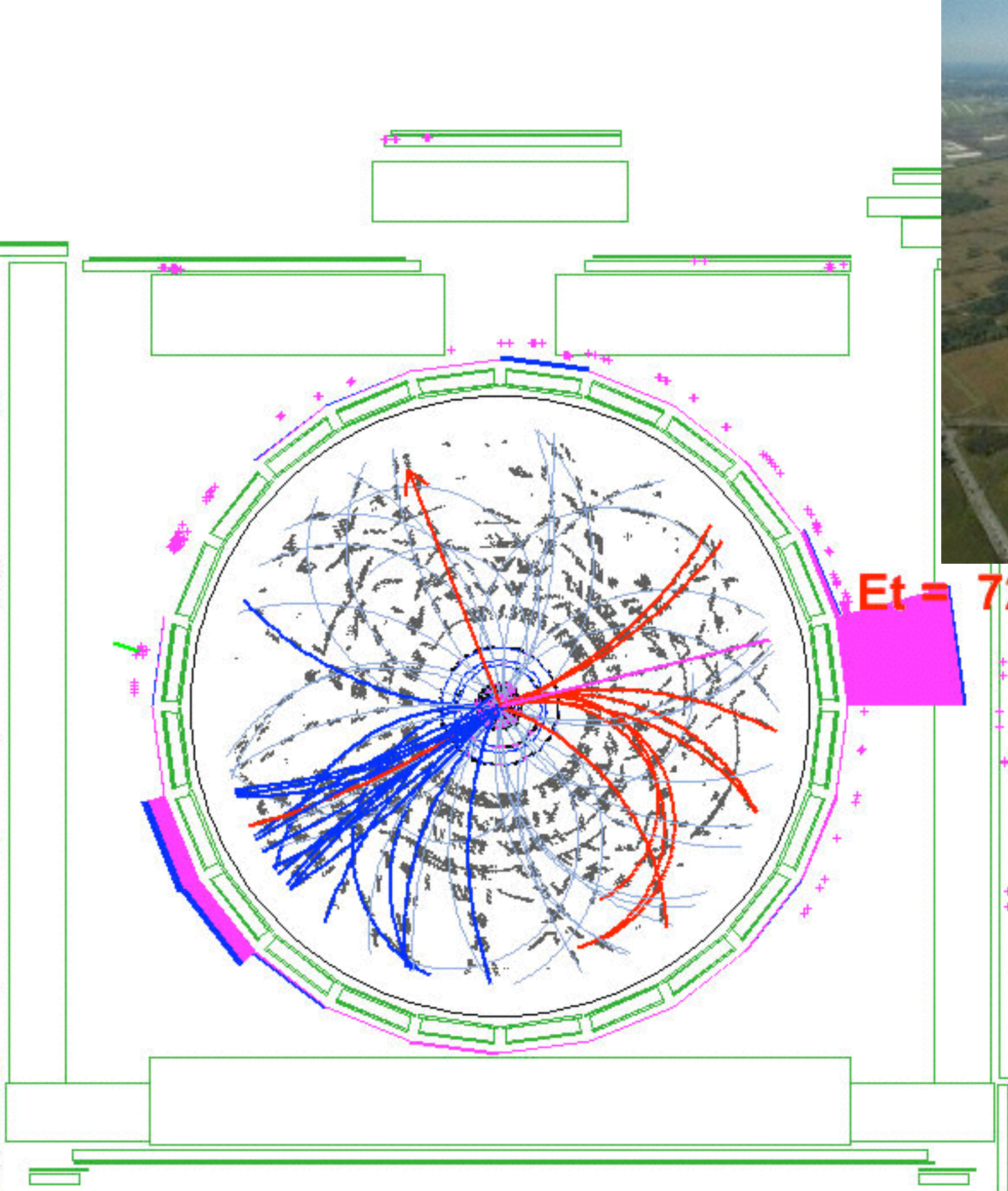


Richard Feynman

Describes physics at short distances, and high energies

Like at Fermilab





Et = 72

Are there new laws governing nature  
at even shorter distances?

